Supplement to Greek Qabalah II

by

Frater Apollonius

4°=7□

A.˙.A.˙.

Of Geometry

The story that Ptolemy, the king of Alexandria, asked Euclid if there was a royal road to geometry, to which he replied there was none; is both problematic and emblematic. While Proclus reported this about 450 AD, over six hundred years later, a markedly different version of this story comes from Stobaeus. Stobaeus (late 5th century AD) states that it was Alexander the Great (356-323 BC) who asked this question of another mathematician by the name of Menaechmus. But, the second version of this story is even less reliable than the first. Consider that Alexander was the private pupil of Aristotle, who had been Plato's student. Plato's Academy, in which Aristotle studied for over twenty years, reportedly proclaimed the following over its entrance: "Let no one untrained in Geometry enter here." Now, this tale, too, may be spurious. But one also should note that Plato started a work Epinomis, posthumously finished by Philippus of Mende, which laid out the appropriate mathematical curriculum for an ideal head of state.

The point here is that Alexander, with the benefit of Aristotle's teaching, would have at least received a primary introduction to the elements of geometry (It is said by Proclus that Hippocrates was the first to compile Elements, thus preceding Euclid), and in view of his training, would have never have asked such a question in the first place. This point is further supported by the following passage from Plutarch's life of Alexander The Great (1st Century AD) that goes on to quote from a letter actually authored by Alexander:

"It seems clear that Alexander was instructed by his teacher not only in the principles and ethics of politics, but also in those secret and more esoteric studies which philosophers do not impart to the general run of students, but only by word of mouth to a select circle of the initiated. Some years later,..., he learned that Aristotle had published some treatises dealing with these esoteric matters, and he wrote to him in blunt language and took him to task..."

The introductory quote is also highly emblematic of the role that mathematics, and specifically geometry, would come to play in the development of Greek civilization. While the Greeks did not have a monopoly on the subject, they were able to take mathematical thinking to an entirely new level and into an unique direction to which much is owed. But, the history of Greek mathematics, which stretches back to 600 BC, abounds with such problematic anecdotes, which is why during the past hundred years, it has become fashionable to dismiss much of it out of hand. But the naysayers miss the mark in two key respects.

The first is that the history of mathematics, apocryphal tales and all, was to have a profound effect on the Middle Ages and the Renaissance, a time not just of discovering the new but of recovering ancient learning and lore. Those who rediscovered the beginnings of mathematics through their study of Plato, Plutarch, Pliny, Iamblichus and Proclus, drank in every word.

The second brings us back to Plato. Plato, who taught Aristotle, was himself taught by Socrates. Plato's dialogues were, in fact, Socratic dialogues that Plato formalized through transcription into philosophical arguments. What the reader witnesses though these dialogues is a great oral tradition that in antiquity was a primary means of imparting instruction and knowledge. Texts were used and available, but there was also an esoterical oral tradition central to teaching mathematics and philosophy which is clearly indicated by Plutarch's reference. This tradition,

which as will see, dated back to the time of Pythagoras, must have continued well after the transmission of Socratic ideas through Plato's written dialogues and lasted at least until Justinian closed Plato's Academy in 529 AD. Elsewhere, such as Alexandria, it may have continued even longer. Hence, an oral tradition of teaching mathematics may have extended an additional eight hundred years after Plato's death. This being so, some of the documentation that comes to us from much later sources such as Nicomachus, Iamblichus and Proclus in the late first century, and from the third and fifth centuries, may well contain ideas culled from this oral teaching as well as preceding texts that are long lost, a tradition that was possibly diminished yet still extant. This oral tradition would eventually dim with the advent of a larger historical decline. But one can well imagine that at one point, a standard work such as Euclid, would have been taught in conjunction with a parallel verbal curriculum.

An inherent problem obtains with any oral tradition. That problem is entropy. With time, distortion and loss of energy alters the message being transmitted. However, such effects were not lost on the Greeks or other ancient cultures. Precautions were taken, two of which were initiation and secrecy. Mnemonic devices (poems) and riddles were also used. Furthermore, over time the encoding of the oral message could be changed to preserve the value of the information. In the two versions of the tale cited earlier, similar information is conveyed through two different contexts. The first had the conversation occurring between Euclid and Ptolemy Soter, the second, between Alexander and Meneachmus. However, the exchange in each case is identical: Question: "Is there a royal road to Geometry?" Answer: "No. There is no short cut!"

The history of Greek mathematics can be divided into roughly four periods. *(fig. 1)*

GREEK MATHEMATICS

THE AGE OF THALES AND PYTHAGORAS

General History	Philosophers and Historians	Mathematicians and Astronomers
610 B.C. The beginning of the New Babylonian empire 540 B.C. The beginning of the Persian empire	Milesian school: Thales Anaximander Anaximenes	585 B.C. Thales 550 B.C. Pythagoras Anaximander
500 B.C. Ionian revolt 480 B.C. Persian wars 450 B.C. Pericles 420 B.C. Peloponnesian war	Heraclitus The Eleatics 450 B.C. Anaxagoras Herodotus 430 B.C. Atomists 410 B.C. Thucydides	500 B.C. Hippasus 500 B.C.–350 B.C. Pythagoreans Anaxagoras Oenopides 430 B.C. Hippocrates Democritus Theodorus
370 B.C. Epaminondas	Socrates †399 Plato Heraclides of Pontus Aristotle Eudemus	390 B.C. Archytas Theaetetus 370 B.C. Eudoxus Callippus Hicetas 350 B.C. Menaechmus Dinostratus Autolycus
333 B.C. Alexander the Great Hellenism	Stoics	300 B.C. Euclid 280 B.C. Aristarchus 250 B.C. Archimedes 240 B.C. Eratosthenes Nicomedes 210 B.C. Apollonius 150 B.C. Hipparchus
60 B.C. Julius Caesar 1 A.D. Augustus 400 A.D. Migration	Neo-Pythagoreans Neo-Platonists: Proclus	60 A.D. Heron 100 A.D. Menelaus 150 A.D. Ptolemy 250 A.D. Diophantus 320 A.D. Pappus

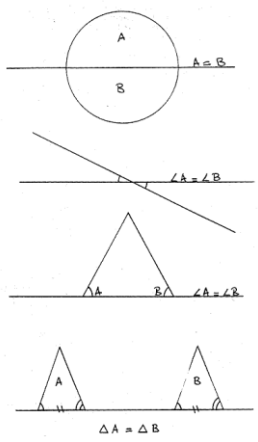
(fig. 1)

The first is the age of Thales and Pythagoras (600 BC to 400 BC). The second is known as the Platonic Age (400 BC to 300 BC); the third, the Alexandrian Age (300 BC to 1 AD) and the fourth, the Ptolemaic Age or post-Euclidean Age (1 AD to 300 AD).

Thales of Milete (585 BC) was said to be the first of the seven sages to arise as the Greek polis emerged from a former feudal system. He was a statesman, philosopher, mathematician and astronomer who at some point traveled the Mediterranean basin in search of knowledge and who returned with extensive empirical observations from the civilizations of Babylon and Egypt. Thales then organized some of this information into the beginnings of a formal mathematical system. He is, as a result, regarded as the father of Greek mathematics.

Thales was famous in antiquity for predicting a total solar eclipse that occurred on May 28, 585 BC. Herodotus reports that this occurred during the battle of the Halys between the Medes and Lydians. At the time the Ionians, Thales was Ionian, were allied with the Lydians. The battle came to a sudden stop, as day turned to night, and the cessation of hostilities led to a lasting peace between both sides. This story is also reported by Diogenes Laertius, who states that Xenophanes (philosopher and poet, c. 570-490 BC) was greatly impressed by this feat. Thales, who had spent time studying astronomy in Babylon, may have returned with intimate knowledge of astronomical observations collected over many centuries by the Babylonians. They may have included certain patterns in solar and lunar eclipses, one being that a solar eclipse will occur 23 1/2 synodic months after a total lunar eclipse. Thales may have predicted the eclipse by counting off the months between events.

Thales is credited with at least four principal geometrical discoveries. He was the first to observe that a circle is divided into two equal parts by its diameter. He also recognized the equality of base angles in the isosceles triangle and observed that when two straight lines intersect each other, the opposing angles are also equal. He provided the definition of congruence which states that any two triangles with one similar side and two similar angles are equal. (fig. 2)



(fig. 2)

There may be other achievements that can be attributed to him, but certainly one of the most important is that he was Pythagoras' teacher.

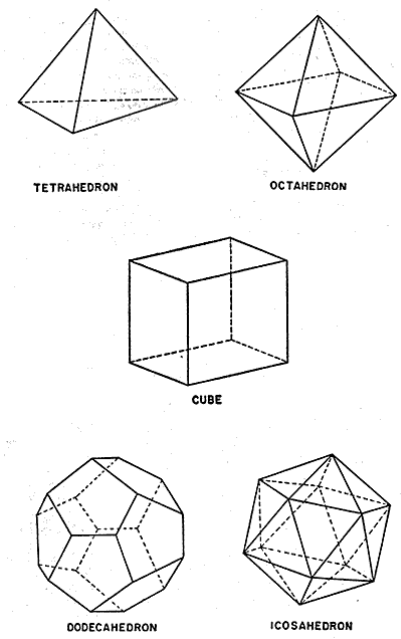
Of Pythagoras

Proclus who headed the Neoplatonic school of Athens during the fifth century AD, wrote a summary of the history of Greek mathematics known as the Eudemian Summary. It is believed that his main source was a prior history authored by Eudemus, one of Aristotle's pupil's, from the third century BC. In his summary, which was part of a commentary on the first book of Euclid, he clearly defines the birth of geometry and the traditional succession of Thales and Pythagoras:

"Since it behooves us to examine the beginnings both of the arts and of the sciences with reference to the present cycle of the universe, we say that according to most accounts geometry was first discovered among the Egyptians, taking its origin from the measurement of areas. For they found it necessary by reason of the rising of the Nile, which wiped out everybody's proper boundaries. Nor is there anything surprising in that the discovery both of this and of the other sciences should have its origin in a practical need, since everything which is in process of becoming progresses from the imperfect to the perfect. Thus the transition from perception to reasoning and from reasoning to understanding is natural. Just as exact knowledge of numbers received its origin among the Phoenicians, by reason of trade and contracts, even so geometry was discovered among the Egyptians for the aforesaid reason.

Thales was the first to go to Egypt and bring back to Greece this study; he himself discovered many propositions, and disclosed the underlying principles of many others to his successors, in some cases his method being more general, in others more empirical. After him Amerstius, the brother of the poet Stesichorus, is mentioned as having touched the study of geometry, and Hippias of Elis spoke of him as having acquired a reputation for geometry. After this Pythagoras transformed this study into the form of a liberal education, examining its principles from the beginning and tracking down the theorems immaterially and intellectually; he it was who discovered the theory of proportionals and the construction of the cosmic figures."

The theory of proportions encompasses the fundamental discovery of ratio in music, mathematics and geometry and will be further enlarged upon below. The three cosmic figures attributed to Pythagoras are the cube, the pyramid and the dodecahedron. To these three, the octahedron and icosahedron were later added by Theaetetus.41 All five came to be known as the Platonic solids or figures and were distinguished by their exact circumscription within a perfect sphere. In Book XI of the Elements, Euclid defines them as follows: (fig. 3)



(fig. 3)

Definition 25: A cube is a solid figure contained by six equal squares.

Definition 12: A pyramid is a solid figure, contained by planes, which is constructed from one plane to one point. (This open ended definition allows for the construction of two shapes with either square or triangular bases.)

Definition 28: A dodecahedron is a solid figure contained by twelve equal, equilateral and equiangular pentagons.

Definition 26: An octahedron is a solid figure contained by eight equal and equilateral triangles.

Definition 27: An icosahedron is a solid figure contained by twenty equal and equilateral triangles.

Pythagoras was from Samos, a Greek island in the Peloponnese off what was then the coast of Ionia (modern Turkey). Historians believe he was born about 560 BC and lived some eighty years (480-490 BC) Like Thales he is thought to have traveled extensively throughout Egypt and Babylonia at an early age, and later, to the other islands of the Peloponnese. In either 530 or 520 BC, he left Samos for Kroton in southern Italy, at that time part of Magna Graecia. There in Kroton, he founded a powerful religious and philosophical sect whose influence extended to neighboring Greek cities of southern Italy and Sicily. In Kroton, the ruling patriarchy came to view the sect as a threat and, in 500 BC, forced Pythagoras and his followers to move to Metapontum, farther up the coast in the heel of Italy, where he died about 480 BC. Later, widespread political upheaval, during which the Pythagoreans were persecuted, forced the sect to the Greek mainland. During this period the group split into two opposing camps, the 'listeners'

and more advanced initiates known as 'scientists-mathematicians'. The former embraced Pythagorean teachings as dogma, while the latter believed in an evolving interpretation of Pythagorean thought. The sect is believed to have subsisted in Phleius and Tarentum until the middle of the fourth century BC.

Two later Greek historians who did much to foster Pythagoras' influence were Iamblichus and Porphyry. Iamblichus authored On the Pythagorean Life, a colorful account of Pythagoras and his followers which was preceded by Porphyry's shorter biography. Born in Syria in the mid-third century AD, Iamblichus is said to have come from a landowning family that had the means to educate him in Antioch and Alexandria. He later worked in Sicily or Rome, where he came under the influence of Porphyry (about 232-303 A.D), a pupil of Plotinus who had written a Life of Pythagoras and an anti-Christian polemic. Both men were instrumental to a Greek revival which sought to challenge the increasing hegemony of Christianity in the third century. Although historians today cast a great deal of doubt upon the reliability of these two authors, there is little question that their writings shaped early scholastic views concerning the development of mathematics.

The main problem with Iamblichus is that he often contradicts himself and is at times inconsistent with the facts. However, from a modem perspective, this is not reason enough to cast doubt over the whole of his account, for a great deal of his biography is indeed culled from very valuable first sources.44

Iamblichus is much clearer than Proclus about the association between Thales and Pythagoras:

"Thales had helped him [Pythagoras] in many ways, especially in making good use of time. For this reason he had renounced wine, meat and even earlier large meals, and had adjusted to light and adjustable foods. So he needed little sleep, and achieved alertness, clarity of soul, and perfect and unshakable health of body. Then he sailed on to Sidon, aware that it was his birthplace, and correctly supposing that crossing to Egypt would be easier from there. In Sidon he met the descendants of Mochos the natural philosopher and prophet [who was believed to have originated Atomism], and the other Phoenician hierophants [priest and keeper of sacred mysteries], and as initiated in all the rites peculiar to Byblos, Tyre and other districts of Syria. He did not, as one might unthinkingly suppose, undergo this experience from superstition, but far more from a passionate desire for knowledge, and as a precaution lest something worth learning should elude him by being kept secret in the mysteries or rituals of the gods. Besides, he had learnt that the Syrian rites were offshoots of those of Egypt, and hoped to share, in Egypt, in mysteries of purer form, more beautiful and more divine. Awestruck, as his teacher Thales had promised, he crossed without delay to Egypt, conveyed by Egyptian seamen who had made a timely landing on the shore below mount Carmel in Phoenicia,... "

Iamblichus then tells of how the sailors thought they might draw some benefit from their passenger by selling him into slavery. However, Pythagoras avoids this fate through correct composure and disposition. He continues:

"From there [Egypt] he visited all the sanctuaries, making detailed investigations with the utmost zeal. The priests and the prophets he met responded with admiration and affection, and he

learned from them most diligently all that they had to teach. He neglected no doctrine valued in his time, no man renowned for understanding, no rite honored in any region, no place where he expected to find some wonder. So he visited all the priests, profited form each one particular wisdom. He spent twenty-two years in the sacred places of Egypt, studying astronomy and geometry, and being initiated - but not just on impulse or as the occasion offered - into all the rites of the gods, until he was captured by the expedition of Kampyses and taken to Babylon. There he spent time with the Magi, to their mutual rejoicing, learning what was holy among them, acquiring perfect knowledge of the worship of the gods and reaching the heights of their mathematics and music and other disciplines. He spent twelve more years with them, and returned to Samos, aged by now about fifty-six."

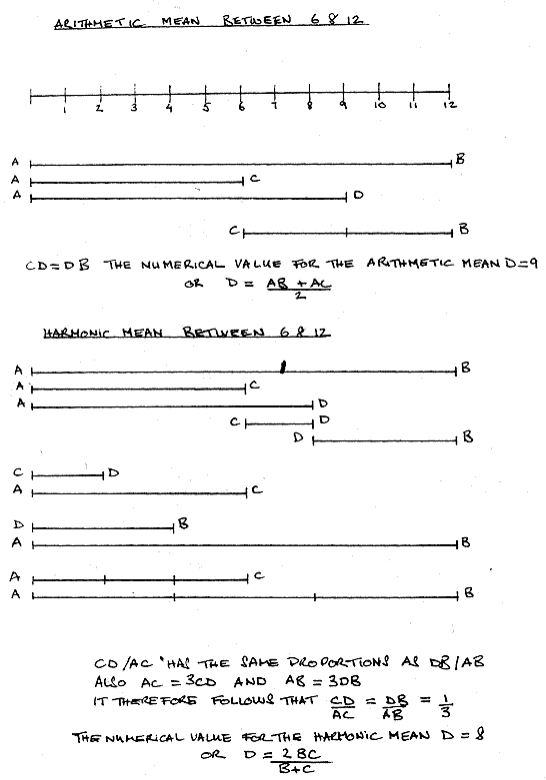
Upon returning to Samos, Pythagoras acquired his first disciple and resumed his travels around the Greek Islands and the mainland seeking knowledge in Delos, Sparta and Crete. At some point, under circumstances related by Iamblichus, he had a revelation:

"He was once engaged in intense thought about whether he could find some precise scientific instrument to assist the sense of hearing, as compass and ruler and the measurement of angles assist the sight and scales and weights and measures assist touch. Providently, he walked passed a smithy, and heard the hammers beating out the iron on the anvil. They gave out a melody of sounds, harmonious except for one pair. He recognized in them the consonance of octave, fifth and fourth, and saw that what lay between the fourth and fifth was in itself discordant, but was essential to fill out the greater of the intervals."

This led him, Iamblichus writes, to a set of experiments with sound and instruments: "And thus he discovered the sequence from lowest to highest note which proceeds by a kind of natural necessity in the diatonic scale. He also articulated the chromatic and enharmonic scales from the diatonic..."

Pythagoras thereby discovered the relationship between musical harmony and mathematical ratio, which he then introduced into geometry. Reportedly, Pythagoras stretched a string over a straight edge, dividing it into twelve parts. On pressing the string at given intervals or shortening the string from 12 to 6 parts, or from 12 to 8 or 9 parts, he obtained tones that were consecutively an octave, a fifth and a fourth higher. He also found that these tones invariably matched the ratios of 2- to-1, 3-to-2 and 4-to-3. This discovery also led Pythagoras to the definition of three kinds of mean: arithmetic, harmonic and geometric.

What was so significant about this? An arithmetic mean, commonly known as an average, is a simple enough idea. It is the difference in two values, the combination divided by two, or mid-point. More important, it is also a point of equality. With an harmonic mean, by contrast, the point of equality is expressed through a common ratio. (fig. 4).



(fig. 4)

The mean was therefore seen as a means to bring opposing ends into harmony. Similarly, on a grander scale, the cosmic order resides in constituent elements that are also in dynamic order. Pythagoras understood this as an universal ordering principle whose mathematical expression was the very essence of existence. The mean as a point of equality and harmony was his fundamental insight to the logic of geometry. By exploring ratio, equalities are established and the underlying dynamic of geometrical structure is revealed.

This notion was best expressed many years later, in a dialogue of Eratosthenes, a major contributor to mathematics and geometry of the Alexandrian era. In The Platonicus, an imagined conversation between two mathematicians, Pappus and Theon, Pappus is given to say:

"Proportionality is composed from ratio and equality is the origin of all ratios. Geometric mediety (mean) indeed has its first origin in equality; it established itself and also the other medieties (means). It shows us, as says divine Plato, that proportionality is the source of all harmonies and ordered existence...."50

At some point, Iamblichus says, Pythagoras decided that his civic duties on Samos interfered with his practice of philosophy:

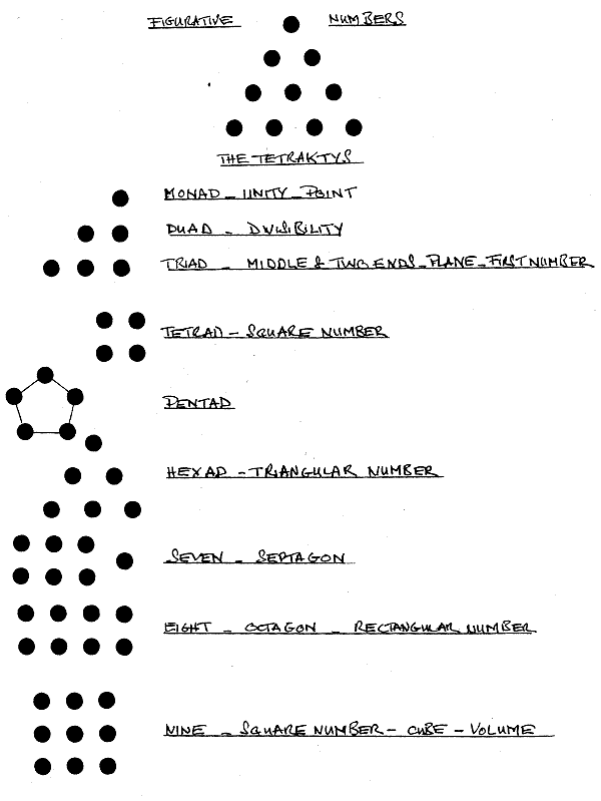
"[So] he left for Italy, resolved to take as his homeland a country fertile in people who were well-disposed to learning. On his first visit to the famous city of Kroton, he made many disciples (it is reported that he had there six hundred people who were not only inspired to study his

philosophy, but actually become 'coenobites' according to his instructions. These were the students of philosophy: the majority were listeners, whom the Pythagoreans call 'acousmatic.'"

By this time, Pythagoras and the Pythagoreans had a central doctrine based on the principles outlined above and supported by various teachings and an ascetic way of life. Central to this ideology was the tetraktys, the initial number series 1,2, 3, 4. This series was arranged in the form of an equilateral triangle of dots which added to the number 10. 1 stood for point, 2 for line, 3 for plane and 4 for solid. The numbers totaled 10, a divine, universal number. In other words, God was number and the universe consequentially rational. The first four numbers were seen to generate the harmonic ratios, linking music with the cosmos, and thus also generating the music of the spheres.

While Iamblichus provides the most complete biography of the Pythagorean movement, it is a prior author of the 1st Century AD, Nichomachus of Gerasa, who most thoroughly summarizes Pythagorean mathematics. Originally from Geresa in Palestine, Nichomachus is thought to have written several books, including an introduction to geometry, a work on astronomy and possibly a Life of Pythagoras that may have been a reference for Iamblichus and Porphyry a few centuries later. None of these have survived. What has is his Manual on Harmonics, a highly Pythagorean work, and his Introduction to Mathematics, which gives a complete explanation of Pythagorean arithmetic and its principles of number, proportion and ratio.

The second book's passages about Pythagorean number theory make abundantly clear how the early Greeks achieved the insights necessary to develop geometry. They devised a new number system which expressed numbers in a highly visual and physical form, through the arrangement of dots. (fig. 5)



(fig. 5)

While 1 is expressed as a unit, and 2 as two units, three is a triangle, four is a square, five is a pentagon, six a hexagon, seven a septagon, eight an octagon. Nine is another square, and ten is a large equilateral triangle with a three-unit side. From this, it followed that higher numbers, those above ten, could be grouped by shapes derived from the original series. Thus, there were square numbers, triangular numbers and pentagonal numbers. This same sequencing could also be used to express a third dimension, in which numbers represented cubes, pyramids and other solids.

With these essentially geometrical identities came a host of symbolic meanings that very quickly became bases for a highly evolved symbolic system, or numerology. This began with the numbers 1 and 2, which were not understood to be numbers per say but expressions of being. One symbolized sameness and unity, 2 represented opposition and divisibility. True numbering commenced only with 3, for 3 had a beginning, middle and end.

Thus, mathematics was a preexisting pattern in the mind of the creator, who in the act of creation reflected this pattern. Of course, with such associations, God or mind were associated with the monad, as were Apollo and the sun. The dyad such was associated to certain properties such as equality and deficiency, and the deities Zeus, Artemis and Aphrodite. The triad was the marriage number, through the combination of the odd and the even, masculinity and femininity. This, too, was also the number for Hecate and Athena. Four, the tetrad, reflected the musical harmonies and was associated with Hermes and Dionysus. The pentad, 5, connoted the heavens and was the number for justice. Six, the hexad, was a perfect number, being the product of 2 and 3, the odd

and even and its own denominators. The sphere and the soul shared its qualities. Seven, an odd prime number, neither generating or generated, was associated with virginity, Athena and the Moon. Eight was unlucky. Nine connoted boundary, because after it all numbers repeated themselves. As an expression of cube (3x3 x3) it was associated with Rhea-Cybele, as well as Hera, Prometheus and Ares. Which brings us to ten.

The decad, embracing all numbers and all numerical forms, was deemed the perfect number. It was the sum of the tetrad elements (1 + 2 + 3 + 4Ñ10) and represented the tetraktys. The Pythagoreans viewed it as the All, the Cosmic, the Universal, Sun, Memory and unending God. It was the number they swore by. Needless to say, the wealth of associations often contradicted each other and were subject to mutation and permutation.

To mathematics, music was added to train the mind and soothe the soul, as Iamblichus explains:

"He no longer used musical instruments or songs to create order in himself: through some unutterable, almost inconceivable likeness to the gods, his hearing and his mind were intent upon the celestial harmonies of the cosmos. It seemed as if he alone could hear and understand the universal harmony and music of the spheres and of the stars which move within them, uttering a song more complete and satisfying than any human melody, composed of subtly varied sounds of motion and speeds and sizes and positions, organized in a logical and harmonious relation to each other, and achieving a melodious circuit of subtle and exceptional beauty."

Hence, the harmonic ratios governed the music of the spheres and governed astronomical theory. This theory held that the Earth, the Moon and the stars, which were perfect spheres, along with the planets rotated around a central, invisible fire. Moreover, all distances were understood to have a harmonic proportion or ratio. Both these ideas would greatly influence scientific thinking many centuries later.

The Pythagoreans believed in reincarnation, the transmigration of souls. They did not eat meat or practice animal sacrifice. They also, on entering the brotherhood, gave up all private effects and held only communal property. The brotherhood also included a sisterhood of wives and independent women. Iamblichus counts the original Kroton members at 218 men and 17 women.

Initiation was a lengthy process:

"The person he [Pythagoras] had examined was sent away and ignored for three years, to test his constancy and his genuine love of learning, and to see whether he had the right attitude and reputation and was able to despise status. After this, he imposed a five year silence on his adherents, to test their self-control: control of the tongue, he thought, is the most difficult type of self-control, a truth made apparent to us by those who established the mysteries. During this time each one's property was held in common, entrusted to the particular students who were called 'civil servants' and who managed the finances and made the rules. If the candidates were found worthy to share in the teachings, judging by their life and general principles, then after the five year silence they joined the inner circle: now within the veil, they could both hear and see Pythagoras.

Before this they were outside the veil: they never saw Pythagoras and shared his discourses only through hearing, and their character was tested over a long period. If one failed the test, he was given double his property and his fellow-hearers... built a grave-mound for him as if he were dead."

Thus, secrecy was of paramount importance to the Pythagoreans. Unfortunately, it resulted in later historians' great reluctance to attribute anything to Pythagoras or his followers. But secrecy was a practice commonly associated with oral traditions that sought to impart sacred truth, whether in Greece or elsewhere in the ancient world.

A final passage from *On the Pythagorean Life* suggests what the Pythagoreans may have been sensitive about:

"The first man to reveal the nature of symmetry and asymmetry to those unworthy to share the teachings was so much detested, they say, that not only was he excluded from their common life and meals, but they built him a tomb, as if their former companion had left human life behind. Some say the supernatural power took revenge on those who published Pythagoras' teachings. The man who revealed the construction of the 'twenty-angle shape' was drowned at sea like a blasphemer. (He told how to make a dodecahedron, one of the 'five solid figures', into a sphere). Some say his fate befell the man who told about irrationality and incommensurability."

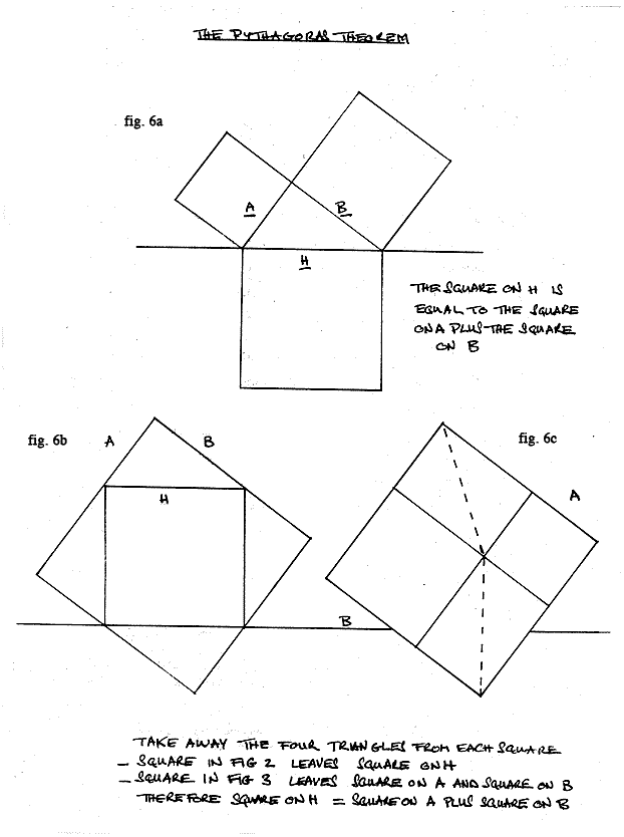
This particular passage, as well as other Greek fragments, point to a certain crisis in the school. It is hard to precisely determine when and how this came about, but clearly at some point this new cosmology was severely shaken. The problem emerged with the examination of the square and the geometric mean.

The objective of the Pythagoreans and of Greek mathematics was to define an ordered cosmos, a rational universe, where everything was in harmony and relationships among elements were expressible in ratios. But there were certain instances where mathematical relationships defied ordinary ratios.

In order to exemplify the problem it is first necessary to turn our attention to two other points. The first is that while geometry evolved into an extremely sophisticated methodology, it was initially predicated on constructions that depended on the use of three very basic tools: a ruler, a set-square and a compass. It is therefore possible to reconstruct the problems the Pythagoreans encountered at an early stage with a number of simple constructions.

The second point is a more sophisticated idea that is traditionally associated with Pythagoras, commonly referred to as the 'Pythagoras Theorem' and identified with Proposition 47 of Book I of Euclid's *Elements*. To be sure, Proposition 47 is a very sophisticated proof which is actually attributed to Euclid (by Proclus) who lived 250 years after Pythagoras. So what can actually be attributed to Pythagoras or his School?

Again, what might be reasonably attributed is a rule which is easily seen with the following constructions. (fig. 6)

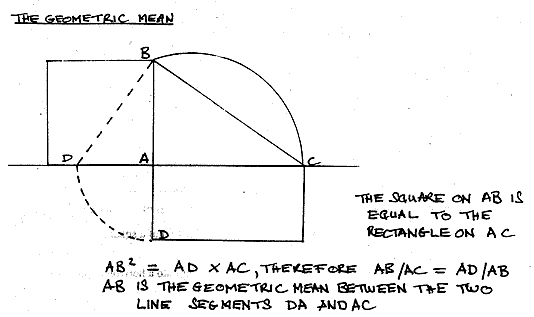


(fig. 6)

Now, the observation that the square on the longer side of a triangle is equal in area to the sum of the squares of the other two sides, was probably taken from the Babylonians or Egyptians, who knew as much in terms of certain right-angled triangles. However, with the Greeks, this property became a rule applied to all right-angled triangles and was finally refined into a proof that covered all right-angled triangles. So what might be attributed to Pythagoras is the establishment of a general rule. The proof would come later.

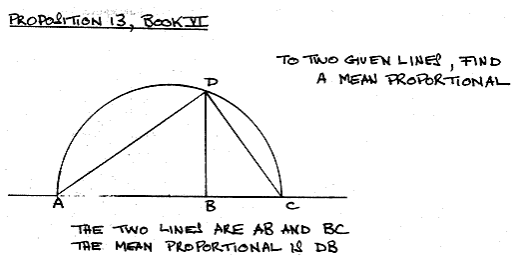
At this juncture we can turn our attention to the discovery of the geometrical mean. Now the mean does have an arithmetic expression which will return to below. However, having explored the origin of the Pythagoras theorem, we can now produce a similar construction which goes along way to also explaining the origin of the third form of mean traditionally associated with the early Pythagoreans.

The construction shows that another rule can be derived between a triangle and the areas formed off its sides. In this instance, it can be shown that the square on one of the sides is equal to a rectangle formed on the adjacent side. (fig. 7)



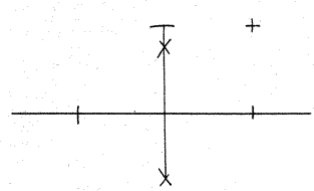
(fig. 7)

This construction which shows the equality of two areas, a square and a rectangle, can now be turned into the following ratio: AB/AC = AD/AB, where AB is the geometric mean between the line segments AD and AC. This rule was also later formalized into a construction in the Elements. Referred to as Proposition 13, Book VI, it shows how to find the geometrical mean between two given line segments. It uses a similar construction in which the proof is the reverse of what is shown. (fig. 8)

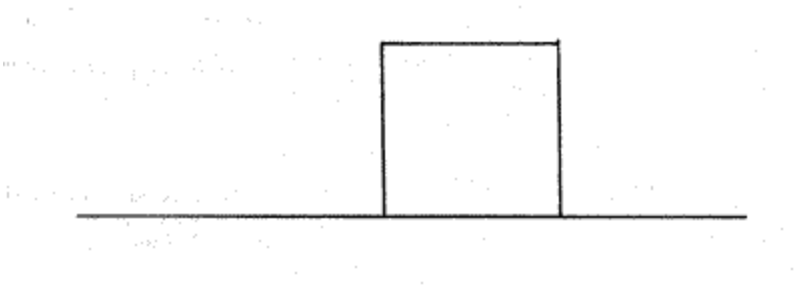


Armed with these two rules, we can now proceed to a number of very simple constructions which involve a compass and the construction of a square:

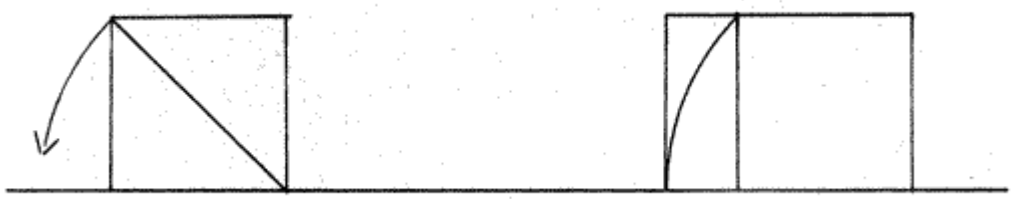
The first construction requires dropping a perpendicular onto a horizontal line. (fig. 9)



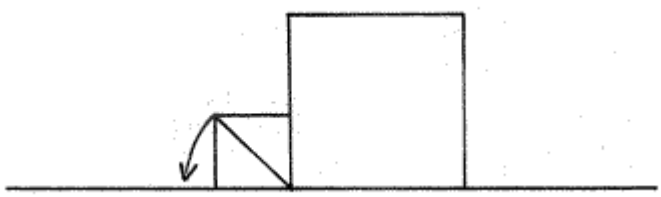
The second requires building a square on the given base line with the help of the perpendicular. (fig. 10)



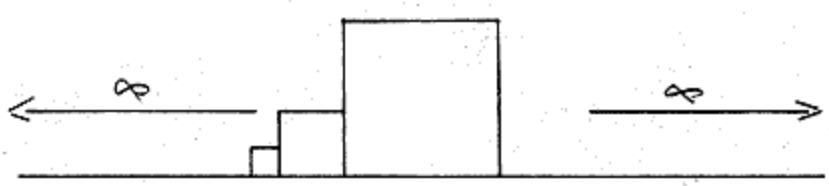
The third requires generating a rectangle with the diagonal of the square. (fig. 11)



The fourth requires generating a new square with the excess of the diagonal over the side of the first square. (fig. 12)

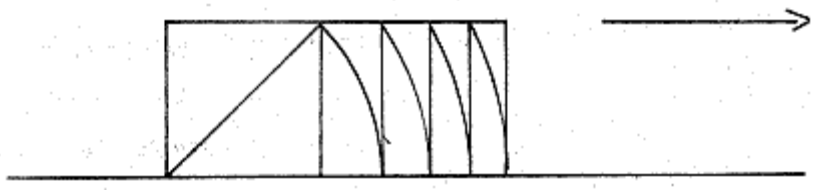


The fifth requires producing a succession of smaller squares with each new diagonal. (fig. 13)

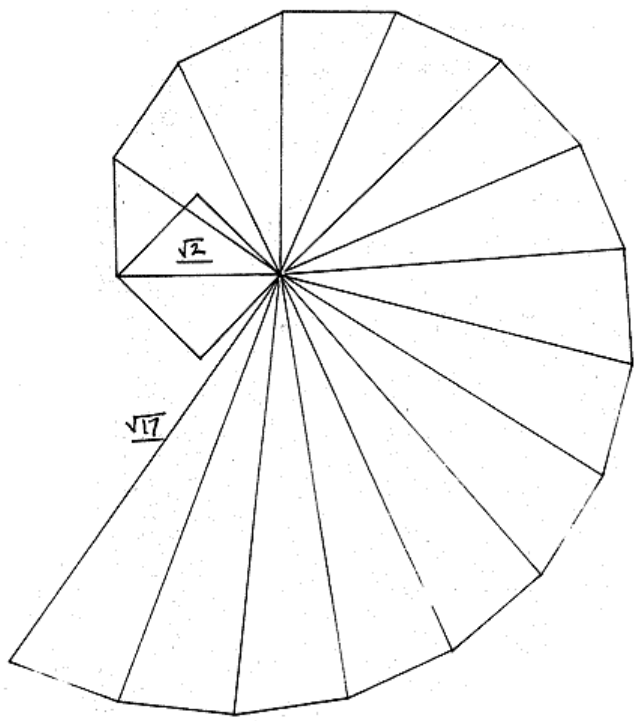


What becomes immediately apparent with the last construction, (fig. 13), is an endless sequence of squares which one may imagine extending for perpetuity in either direction. We have met the ineffable.

In (fig. 14), we return to the initial square and construct a succession of ever-increasing rectangles.56



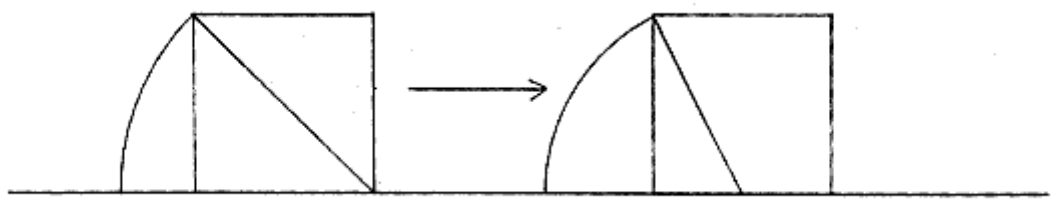
By placing a set square on the first diagonal of the square and marking off a length equal to the side of the initial square, one can produce a spiral. (fig. 15)



(fig. 15)

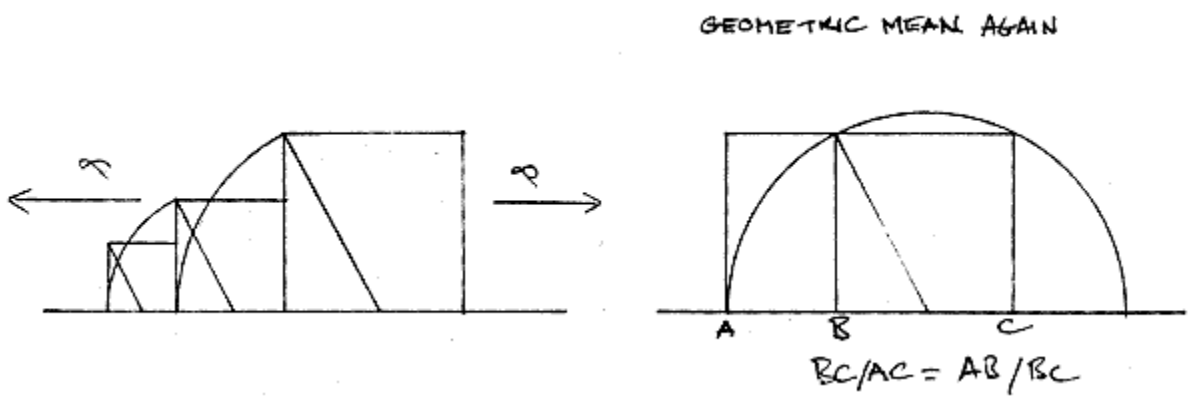
Each spoke becomes the diagonal of each successive rectangle in (fig. 15). 57 The spiral will eventually turn into an endless smooth curve. This too is ineffable.

If we return to our initial square first shown in (fig. 11), which is again shown as (fig. 16a),



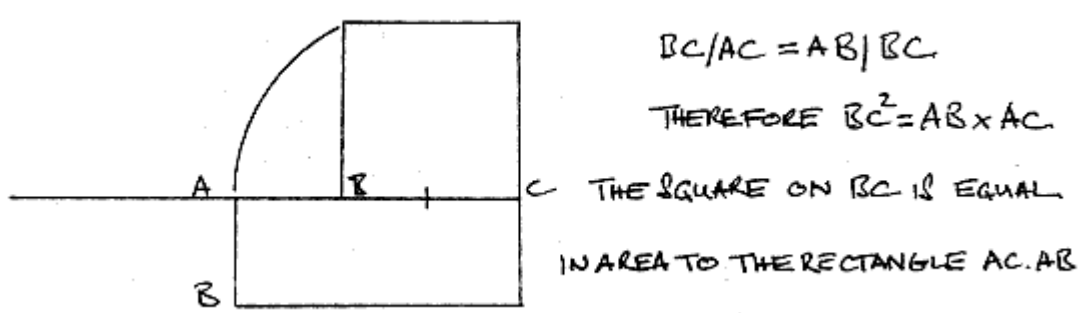
(fig. 16a) (fig. 16b)

and place the compass point at a mid-point on the base line instead of the corner of the square and produce (fig. 16b), this too can be used to generate another sequence of diminishing squares (fig. 17a).



(fig. 17a) (fig. 17b)

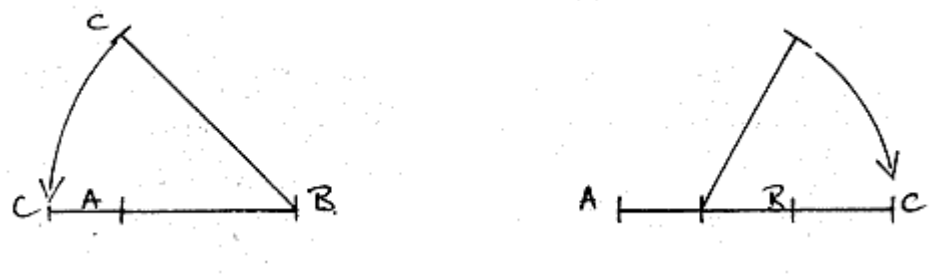
However, (fig. 17b) is similar to (fig. 7) and (fig. 8). It is as if the triangle in (fig. 7) and (fig. 8) has collapsed onto its base line. We have encountered another expression for the geometric mean, so that the square on AB is equal to the rectangle on AC. (Fig 18)



(fig. 18)

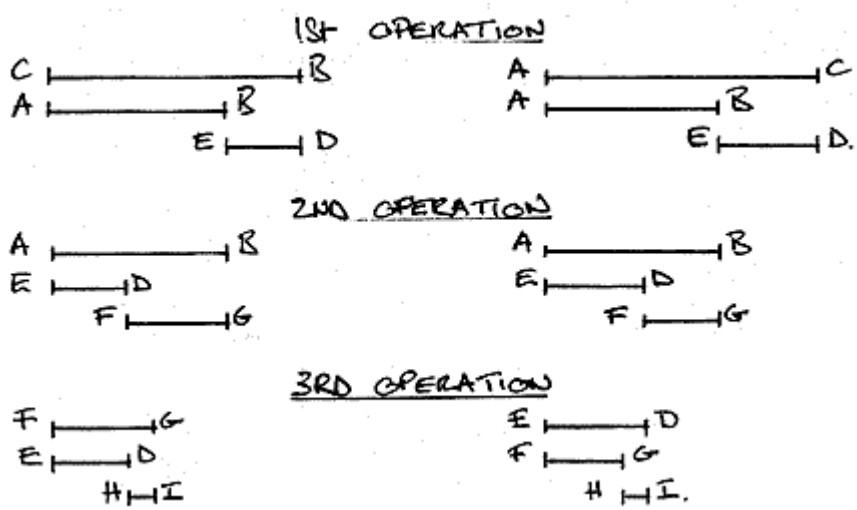
This in turn means that on the line segment AC, we have the following ratio: AB/AC = BC/AB.

What was the explanation for these endless sequences? What was the ineffable that they had encountered? If we return to figs. 16a & 16b, and strip them down, we are left with figs. 19a & 19b.



(fig. 19)

Now if we then set the shorter segment AB from each construction as an unit length, i.e.: AB = 1, and measure it against the longer segment, AC and CB, in both cases we are left with a remainder ED. (fig. 20)



(fig. 20)

If we take the remainder as a new unit length, and measure it off against AB, we are again left with a remainder FG. By the third operation it becomes apparent that the unit length will never quite measure out the remainder. It will never generate a whole integer or fraction (3 or a 1/2.)

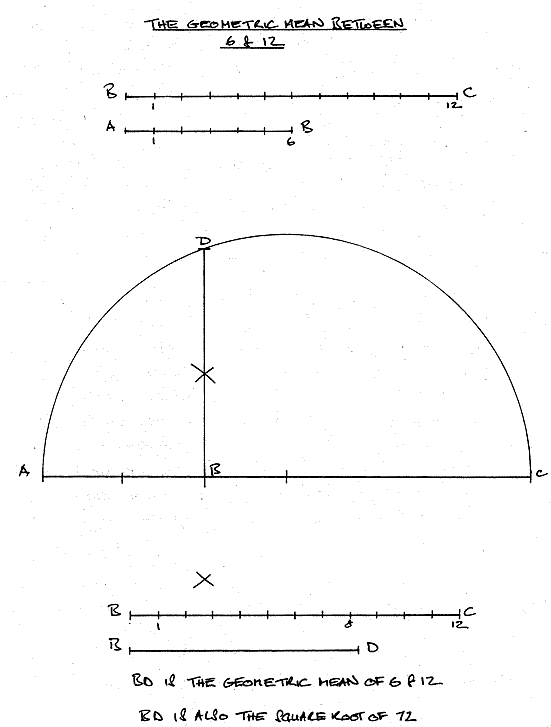
The Greeks had encountered what they would eventually term 'the irrational'. If two lines could not be used to measure each other or did not have a common measure, they were deemed

'incommensurable'. If one line could be used to divide the other into a solid integer or approximate fraction they were said to be 'commensurable'.

It should be noted, if not stressed, that the Greeks did have a very sophisticated theory of numbers or integers, known as the 'theory of the odd and the even,' which allowed for fractions. And oddly enough it is through the 'theory of the odd and even' that a proof was finally devised to show that the side and diameter of a square are incommensurable. This proof came much later and is provided in Aristotle's Metaphysics, Book IX. But numerically, what would be necessary here, is the notion of decimals. But decimals require Arabic numbers which were not introduced until the 13th century by Fibonacci. And, decimals themselves were not explored until the 17th century. Thus, when Archimedes (200 BC) discovered Pi (3.14), he expressed it in terms of two limiting approximations, less than 3 1/7 and more than 3 10/71.

However, at this point we can review a number of things numerically. Reviewing the relationship between the side of a square and its diameter, according to the theorem of Pythagoras, the square of the diagonal's length equals the sum of the squares of the two sides. However, if the sides have a length of 1, then the formula becomes 1 squared + 1 squared = 2 squared, and there is no whole number or rational number that when multiplied by itself is equal to 2. The answer is provided numerically with the figure: 1.414....

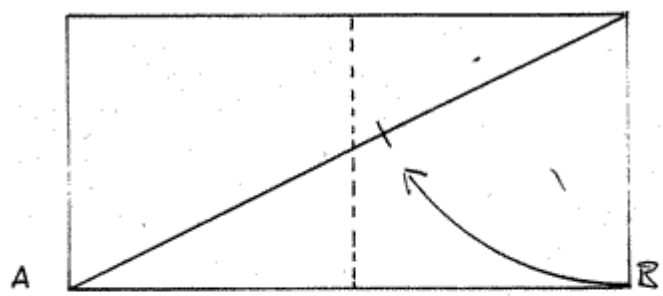
Returning to the extremes 6-12, and the three means, where the arithmetic is given as 9 and the harmonic as 8; the geometric mean is provided by the square root of 6 x 12 = 72, or 8.485.... The geometrical solution is provided in *(fig. 21)*.

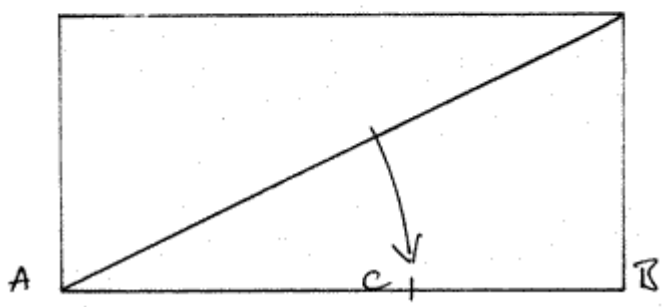


(fig. 21)

In *fig. 17b*, where we encountered an incommensurable expression of the geometric mean, what we really came upon is an unique ration known as the Golden Section.60 The unique feature to this ratio is that the remaining line segment will always be in the same ratio to the prior line segment it is compared to. Thus DC is in the same ratio to AB, as AB is to AC (fig. 20).

The construction in *fig. 17b* is therefore used to extend a given line, where a second segment is added to the first in order to provide the complete ratio. *Figs. 22a & 22b* are now provided to show how a given line can be cut into the same ratio.

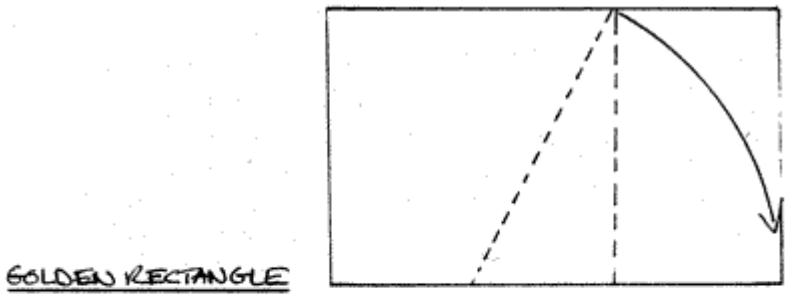


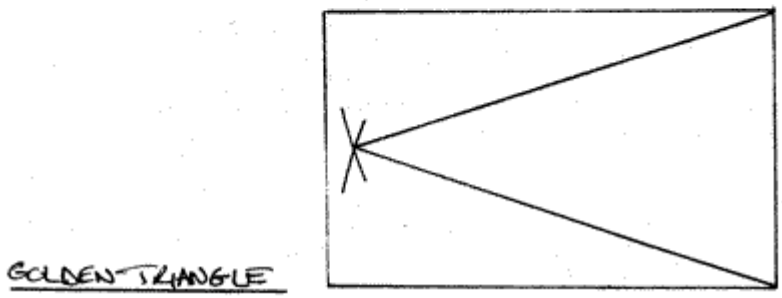


(fig. 22a)

(fig. 22b)

The construction requires building a double square on the line and using the diagonal of the resulting rectangle to find the cut. This too is presented in the Elements as Proposition 30, Book VI. The construction used is provided by *fig. 23* which is similar to *fig. 17* and *fig. 18*. The Proposition states: "To cut a given finite straight line in extreme and mean ratio." Thus the term 'mean and extreme ratio' is also ascribed to the Golden Section. There are two other geometrical shapes that share the same nomenclature: the Golden Rectangle and the Golden Triangle. The rectangle is again *fig. 17b*. The triangle is generated with the diagonals of the Golden Rectangle. *(figs. 23a &23b)*





(fig. 23a)

(fig. 23b)

The Golden section is associated with the Pythagoreans on two counts. The first association is literary through the adjective 'Golden'. The second is again geometrical and through the Pentagram.

There is a poem that comes down to us known as the *Golden Verses*. It provides the initiate with the exoteric tradition of the Pythagorean Community. The poem blends popular wisdom with the esoteric insights that are promised to the student who follows its Golden Path. Its transcription is dated about 350 to 300 BC, but it may have been part of an oral tradition that predates this time. The following excerpt illustrates the point:

Do not welcome sleep upon your soft eyes
before you have reviewed each of the day's deeds three times:
"Where have I transgressed? What have I accomplished? What
duty have I neglected?"
Beginning from the first one go through them in detail, and then,
if you have brought about worthless things, reprimand yourself,
but if you have achieved good things, be glad.
Work had at this, meditate on this, you should passionately
desire this;
this will put you in the footsteps of divine Virtue,

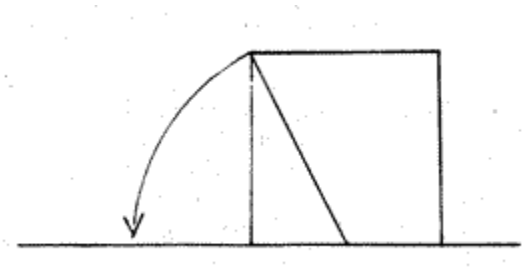
yes, by him who imparted the *tetraktys*,
font of ever-flowing nature. But to work!
and pray to the gods to grant fulfillment.
When you have mastered these things,
you will come to know the essence of immortal gods and mortal men, etc.....

The Pythagoreans were also associated with the pentagram. A pentagon's diagonals form a star-pentagon, or pentagram, which served as a symbol of health, as well as an icon for the sect. Lucian, writing in the 2nd century AD, explains:

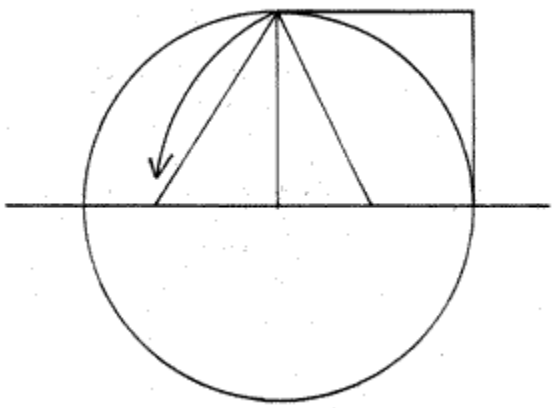
"The divine Pythagoras chose not to leave anything of his own, but if we may judge by Ocellus the Leucaman and Archytas and his other disciples, he did not prefix "Joy to you" or "Do well", but told them to begin with "Health to you". At any rate all his school in serious letters to each other began straightway with "Health to you", as a greeting most suitable for both body and soul, encompassing all human goods. Indeed the pentagram, the triple intersecting triangle which they used as a symbol of their sect [literally, those of the same teaching], they called "Health."

Now, the Golden Section, as we will see, is essential to the construction of the pentagon. There are two basic constructions for the pentagon. The first is non-rigorous. The second rigorous and found in Euclid as Proposition 11, Book IV, and requires the construction of a Golden Triangle which is given in Proposition 10.

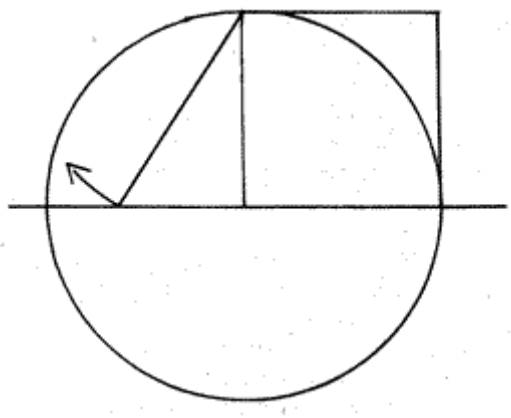
Let us turn to the non-rigorous construction provided in *figs. 24-28*.



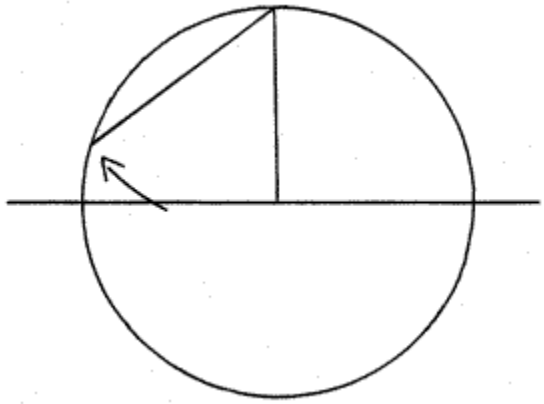
(fig. 24)



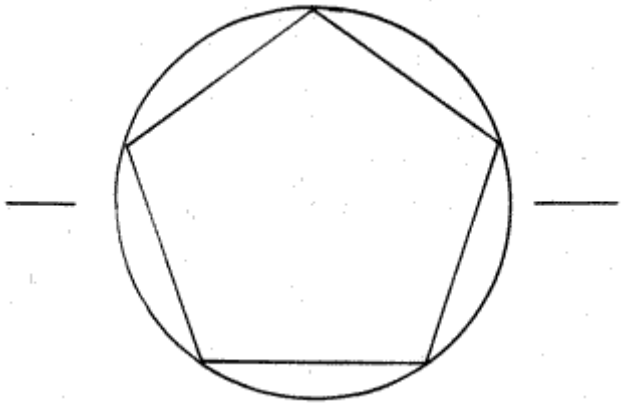
(fig. 25)



(fig. 26)

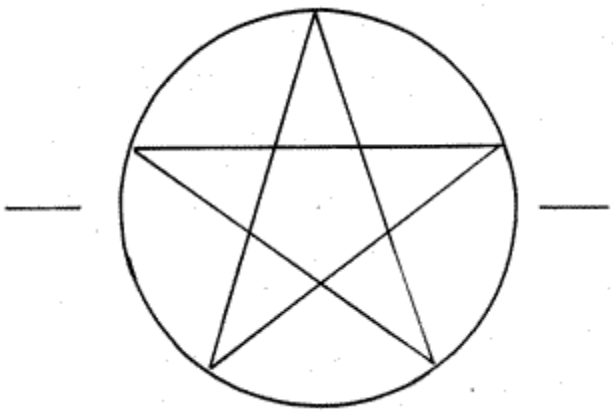


(fig. 27)



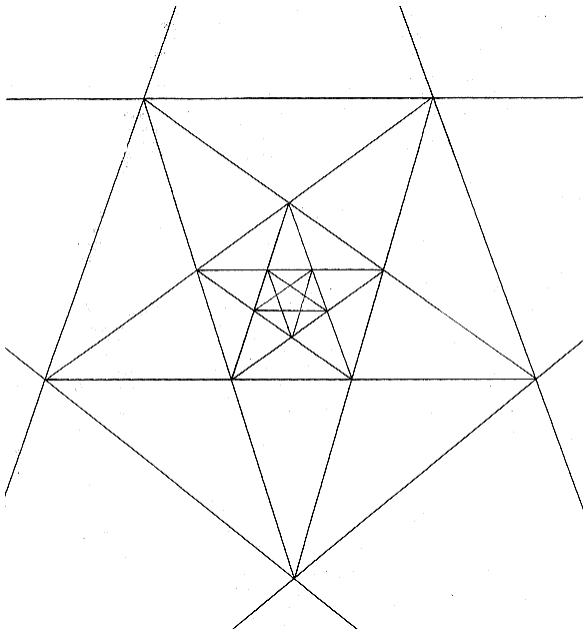
(fig. 28)

Again we start with *fig. 17b*, which we now know to be the Golden Rectangle. The Golden Rectangle is used to generate a diagonal (of a smaller Golden Rectangle) which is then applied to the circumference of the circle. This length is then used to mark-off the sides of the polygon. These same points also provide the apexes for an inscribed star-pentagram.*(fig. 29)*



(fig. 29)

Each figure can be used to generate the other in an endless pattern, thus invoking the ineffable yet again. *(fig. 30)*



(fig. 30)

If we return to Iamblichus' quote about the man who blasphemed for revealing Pythagoras' teaching and set aside "symmetry and asymmetry," we are left with three other revelations: the construction of the dodecahedron, irrationality and incommensurability. The dodecahedron is comprised of twenty sides where each side is the shape of a pentagon. Therefore, as demonstrated, all three issues as well as the geometric mean, are closely related to each other and to endlessly generating geometric forms which invoke the ineffable.

But what is the ineffable? The ineffable is infinity. The early Pythagoreans had discovered infinity. The problem is that infinity does not admit of ratio. This discovery was kept secret till someone revealed it. Iamblichus, does not provide the person's name in this context, and he provides an incorrect name in a later context. The name is provided in other sources as Hippasus.

It would seem that in about 420 BC, or a little earlier, towards 450 BC, there was a schism in the sect, between the acousmatics and mathematicians. The turmoil was probably due to the forced exile of the Pythagoreans from southern Italy because of political rivalries. The 'listeners' wished to uphold the very word of their teacher. The mathematicians wished to explore the process initiated by Pythagoras. At the same time, there was probably another conflict which fell within the same lines between those who wished to teach for a living and those who believed the teaching sacred and reserved for the initiated. This conflict is documented in Iamblichus by a letter from Lysis to Hippasus. Iamblichus misnames Hippasus, Hipparchos. Lysis on fleeing from Italy moved to Thebes. Hippasus later perished at sea.

The letter states:

"You say we should philosophize in public, for whoever comes along. Pythagoras said not, and so you learnt, Hipparchos, in all seriousness. But you did not keep the teaching safe. You had the taste of the Sicilian high living, man, though you should have got the better of it. If you change, I shall rejoice; if not, you are dead. It is right, they say, to keep in memory his commands on divine and human matters, and not to share the goods of wisdom with people whose souls are not remotely purified. It is not right to hand out to chance-met persons what was achieved with so much effort and toil, nor yet expound to the uninitiated the mysteries of the Two Goddesses of Euless - those who do either are equally wrong and impious.

"Think how long a time we spent cleansing the stains which were ingrained in our breasts, until, with the passage of time, we were able to receive his words. As dyers cleanse and treat with a mordant the parts of the garment which need to be dyed, so that the dye will be fast and will never fade or be lost in the wash, so that wonderful man prepared the souls of those who had fallen in love with wisdom, so that he should not be disappointed in one of those he hoped would become good men. He did not purvey false words of the snares which most sophists, working for no good purpose, entrap young men: he knew about divine and human affairs. But those others make his teaching a pretext and do terrible things, hurting young men in the wrong way and of set purpose."

The reason I have brought the reader so far on these matters is because firstly, the Golden Section or Divine Proportion was of particular concern, as was the idea of proportion, to Leonardo Da Vinci. As we shall see, the Golden Section and the tetraktys are key elements of

The Last Supper. Furthermore, the problem of infinity is also re-encountered below, as is Lysis' letter.

Infinity is a spectrum with two extremes. One extreme tends to the infetissimal, the other to an unimaginable magnitude. Looking at *fig. 17a*., the infetissimal tends to the left, infinite magnitude, or infinity to the right. With time, the problem of infetissimals would be tackled by the Greeks. By Aristotle's time, a hundred years or so later, it is a question of open debate as evidenced by Aristotle's own writings. But the question is kept within strict limits. Infinite divisibility is viewed as a potentiality and not an actuality. 'Potentiality' restricts the phenomena to an imagined space. As for the question of infinite magnitude, it is denied.

The problem raised by the irrational line would also, once revealed, be later explored. By the time of Socrates and Plato, at least four mathematicians; Archytas, Theodorus, Theaetetus and Eudoxus, had transformed the impasse into a theory of irrational lines by confining such problems to the realm of geometrical line segments. Most of this work would reappear in a highly formalized fashion in *Euclid*.

Which brings us to the great work itself. As probably apparent, the Pythagoreans' greatest impact lay in a geometrical foundation which eventually led to the *Thirteen Books of the Elements* by Euclid.

Euclid (300 BC), a mathematician, is believed to have elaborated the first formalized system of intellectual human thought. This system would greatly influence a parallel tradition of Greek philosophy and would subsequently shape the course of various Western sciences. Euclid organized his principal work, *The Thirteen Books of The Elements*, around a kind of mathematical alphabet, starting with three basic definitions and then progressing from proposition to proposition, theorem to theorem, element by element. Earlier mathematicians had attempted such works, such as Hippocrates (450 to 430 BC) and Leon (circa 400 BC). But because of the particular rigor of its organization, the *Elements* superseded the others and was the only work of its kind to survive.

With Euclid begins the ascendancy of the Alexandrian School (300 BC - 150 BC), which would come to include such pivotal mathematicians such as Aristarchus, Archimedes, Erastosthenes, Appolonius, Hipparchus and Ptolemy. Only two facts seem to be known about Euclid. The first is that he was born after Plato's pupils, yet he predated Archimedes, who died in 212 BC. The second is that he taught in Alexandria.

Of Alexandria

Crucial to the further development of Greek mathematics and science was Alexander the Great's founding of Alexandria in 332 BC. Alexander stayed only for a few months in the city, after which he never saw it again. The city's development was undertaken by Alexander's viceroy who also arranged that Alexander's body be entombed there.

Situated on the Mediterranean, 12 miles west of the Canopic mouth of the Nile, the city was laid out in the shape of a T, the top of which was a former island made into a peninsular and the trunk of which formed a bridge between the peninsular and another stretch of land that separated the sea from a lake. The former island was known as Pharos, and the city's two large harbors made it the most important port on the Mediterranean. On Pharos stood the largest and most famous lighthouse of antiquity. (Whence the term for lighthouse or beam of light in certain languages.) However, Alexandria's most important feature was its library system, a center for Greek learning that would quickly eclipse Athens. Alexandria's intellectual hegemony lasted well into the Roman era, during which it was the most important Western city after Rome. Whereas Athens became utterly eclipsed by the Rome, Alexandria thrived as a fountainhead of Greek civilization. Its beacon would shine for over six hundred years. 45

Alexandria's first king, Ptolemy Soter, started the city's book collections. His successor, Ptolemy Philadelphia, an avid bibliophile, is thought by some to have acquired what remained of Aristotle's vast collection. Under Philadelphus, two libraries were established in separate buildings. The larger was located in the Bruchium quarter, which with the Museum formed an academy. The smaller was in another quarter called the Serapeum. Philadephus sought out valuable works in every part of the known world and copied them at great expense. The successor, Ptolemy Euergetes, was a ruthless sort who systematically seized all books brought into Egypt, returning only copies of originals to the unfortunate owners. According to the mathematician Erastothanes, who was the head librarian at one point, the Searpeum housed 42,800 papyrus rolls, and there were 490,000 rolls in the Bruchium. Of Alexandria's first five librarians, Zenodotus, Caliimachus, Erastophanes, Appolonius and Aristophanes; Erastophanes and Appolonius were preeminent mathematicians of the age.

The libraries were gradually despoiled as a result of a number of major fires. The first occurred in 46 BC, when Julius Caesar torched the Egyptian fleet in the harbor. The flames swept through the Bruchium, destroying the larger library and other facilities. In the mean time, a rival library had been founded at Pergamon, on the Turkish coast, which flourished despite the monopoly that the Egyptian kings held over the production and distribution of papyrus used in manuscript production. To placate Cleopatra, a few years later, Mark Anthony seized all 200,000 volumes from Pergamon and put them in a new building in the Bruchium. In 273, the main library was again destroyed by fire, on orders from the Emperor Aurelian, after which the Serapeum became the principal library. This, too, was set on fire on an edict of Theodosius in 398 or 390, after which, its remains were pillaged by the Christians. The Serapeum's final destruction is believed to have occurred about 640, when the city fell, following a 14 month siege to the Arab general Amu. On the capture of the great city, Amu reported to the Caliph Omar that the city contained 4000 palaces, 4000 baths and 400 theaters or places of amusement. Amu begged the Caliph to be awarded the Royal Library as a prize. The Caliph replied that if library's books contained

Koranic teachings, they were superfluous, and if they did not, they violated the Koran and should be destroyed. This pronouncement resulted in the distribution of all the books to Alexandria's 4,000 public baths, where for the next six months they served to fuel the fires that warmed the waters.

Of Euclid

Euclid's residence in Alexandria is reported by Pappus (320 AD), who observes that Apollonius spent many years studying with Euclid's pupils in that city. Writing more than a century later, Proclus, after a brief synopsis of the Platonic school, which included Hippocrates, Eudoxus, Menaechmus and Theaetetus; says of Euclid:

"Those who compiled histories carry the development of this science up to this point. Not much younger than these is Euclid, who put together the Elements, arranging in order many of Eudoxus' theorems, perfecting many of Theaetetus', and also bringing to irrefutable demonstration the things which had been only loosely proved by his predecessors. This man lived in the time of the first Ptolemy; for Archimedes, who came immediately after the first Ptolemy, makes mention of Euclid; and further they say that Ptolemy once asked him if there was in geometry a way shorter than that of the elements; he replied that there was no royal road to geometry. He is therefore younger that the pupils of Plato, but older than Eratosthemnes and Archimedes. For these men were contemporaries, as Eratosthenes somewhere says. In his aim he was a Platonist, being in sympathy with this philosophy, whence it comes that he made the end of the whole Elements the construction of the so-called Platonic figures. There are many other mathematical writings by this man, wonderful in their accuracy and replete with scientific investigations. Such are the Optics and Catoptrics, and the Elements of Music, and again the book on Divisions. He deserves admiration pre-eminently in the compilation of his Elements of Geometry on account of the order and of the selection both of the theorems and of the problems made with a view to the elements. For he included not everything which he could have said, but only such things a he could set down as elements. And he used all the various forms of syllogisms , some getting their plausibility from the fist principles, some setting out from demonstrative proofs, all being irrefutable and accurate and in harmony with science."

There is an aesthetic, minimalistic simplicity to Euclid. Starting with three simple statements - there is point, there is line, there is plane - the Elements advance step by step like a Jacob's ladder from the geometry of the triangle, to that of the circle, the parallelogram and the formation of areas. Propelled by the logic of ratios and the theory of irrational lines, Euclid arrives finally at the geometry of the sphere and the five Platonic solids that inhabit three-dimensional space. Sometimes plodding, at times repetitious, each wrung is anchored in an unshakable logic that rises from the elementary to the highly complex, at certain points taking astonishing turns and transformations that flip human perception inside-out and upside down.

One can argue that *Euclid* is a prime example of intellectual abstraction. In fact, many have contended that the Greeks were less concerned with resolving concrete problems than with the advancement of pure logic. This argument is often employed to explain Archimedes' works and attitude. Yet history attests that Archimedes addressed a great many practical problems involving warfare and weaponry. At the same time, while Euclidean mathematics and geometry may

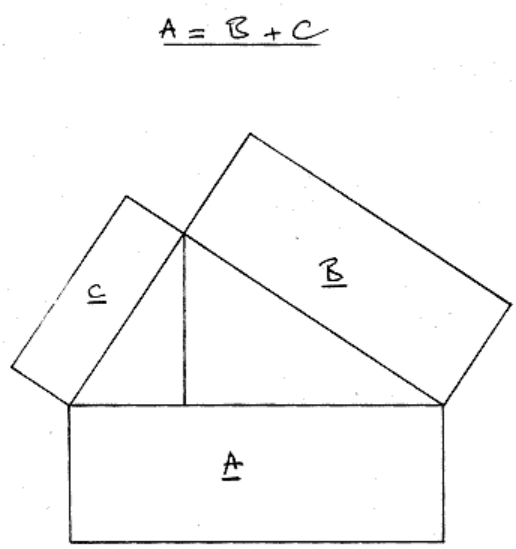
indeed epitomize pure abstraction, the last three books of the Elements lay the groundwork for modern physics. Even though the Greeks fashioned mathematics as an abstract construct, it would become a very potent tool for resolving pragmatic problems.

With *Euclid*, of course, there are no short-cuts. One must take the argument step by step, wrung by wrung, and climb the ladder slowly. Having reached the summit, one finally comprehends why people have scaled this structure for more than twenty centuries. One marvels at the way it affords such certainty in a uncertain universe where the gods are often fickle.

Permeating the structure, moreover, are two fundamental mathematical constructs, to which we have already been introduced. The combination of these would prove vital to the development of trigonometry - the point at which geometry becomes physics.

If the Pythagoreans had provided a general rule for the sides and hypotenuse of a triangle, it would be left to Euclid to present a final proof. The proof comes at the end of Book I, under the form of Preposition 47. Since Book I is comprised of 48 prepositions, the proof appears as a crowning moment in the gradual progression of the Elements of Book I. Later, in Book VI, the theorem is represented in a version of that has wider applications, through an argument that is entirely based on proportion. "If we listen," says Proclus, "to those who wish to recount ancient history, we may find some of them referring this theorem to Pythagoras...But for my part... I marvel more at the writer of the Elements, not only because he made it fast by a most lucid demonstration, but because he compelled assent to the still more general theorem by the irrefragable arguments of science in Book VI."

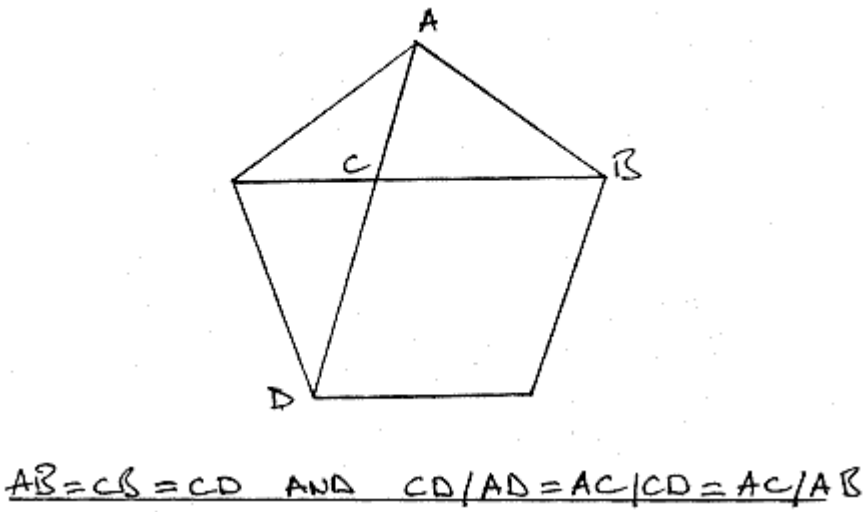
The general theorem in question is Proposition 31, Book VI, which states: "In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides of the right angle. As the figure shows, the construction for the proof is a fusion of the geometrical mean and the Pythagoras Theorem. *(fig. 31)*



(fig. 31)

There are over four hundred theorems in Euclid, and the Pythagoras Theorem is so pivotal that Euclid employs it explicitly thirty-eight times throughout The Elements. It is hard to estimate the number of times it is used implicitly.

The Golden Section reemerges, in a striking coda, in Book XIII, The Elements' final book. The book begins with a group of propositions concerning the Golden Section's properties. It then describes the inscription of the five Platonic solids in a sphere. It is in the last two of these figures, the icosahedron and the dodecahedron, that Golden Section comes into play. And it is in Proposition 8, Book XIII, that identifies its conspicuous and potent role in the pentagon: *If in an equilateral and equiangular pentagon straight lines subtend two angles taken in order, they cut one another in extreme and mean ratio, and their greater segments are equal to the side of the pentagon.*82 *(fig. 32)*



(fig. 32)

Here, with the construction of the Platonic solids within the perfect form of the sphere, Euclidean geometry culminates. The geometry is logical, pure, ethereal and abstract, and one may wonder what, if any, practical value it all has. However, four hundred and fifty years later, in 150 AD, Ptolemy, the last of the great Alexandrian mathematicians, supplied an answer when he mined Book XIII's construction of the pentagon and decagon to complete the first complete table of chords.

The very first theorem in Ptolemy's Almagest, antiquity's final word on astronomy, uses the Golden Section and the Theorem of Pythagoras repeatedly to determine the section of chord subtended by 36- and 72- degree arcs. Using these two values Ptolemy went on to assemble a table of chords from half a degree to 159 1/2 degrees. Not only was this the beginning of trigonometry, it marked the beginning of mathematical astronomy, since with this table of chords it was possible to fix precisely the longitude and latitude of celestial constellations. This development would make it possible to later navigate the oceans and eventually leave the planet.

VARIOUS TEACHINGS

The same author tells us, as I have already mentioned, that he received his doctrines from Themiclea at Delphi. Hieronymus says, that when he descended into the shades below, he saw the soul of Hesiod bound to a brazen pillar, and gnashing it's teeth; and that of Homer suspended from a tree, snakes around it, as a punishment for the things that they had said of the Gods. Those who refrain from commerce with their wives also were punished and that on account of this he was greatly honored at Crotona. Aristippus of Cyrene, in his Account of Natural Philosophers, says that Pythagoras derived his name from the fact of his speaking (agoreuein), truth no less than the God at Delphi (touputhieu).

He used to admonish his disciples to repeat these lines to themselves whenever they returned home to their houses:

"In what have I transgressed? What Have I done? What that I should have done have I omitted?"

He used to forbid them to offer victims to the Gods, ordering them to worship only at those altars which were unstained with blood. He also forbad to swear by the Gods, saying, "That every man ought so to exercise himself as to be worthy of belief without an oath. He also taught men that it behoved them to honor their elders, thinking most honorable that which was precedent inpoint of time; just as in the world, the rising of the sun was more so than the setting; in life, the beginning more so than the end; and in animals, production more than destruction.

Another of his rules was that men should honor the Gods above the geniuses, and heroes above men and of all men, parents were those entitled to more honor. Another, that people should associate with each other in such a way as not to make their friends enemies, but to render their enemies friends. Another was that they should not think anything exclusively their own. Another was to assist the law, and to make war upon lawlessness. Not to destroy or injure a cultivated tree, nor any animal which does not injure man. Modesty and decorum consisted in never yielding to laughter, without looking stern. Men should avoid eating too much flesh, and in travelling should let rest and exertion alternate; that they should exercise memory, nor ever say or do anything in anger, not pay respect to every kind of divination, should sing songs accompanied by the lyre, and should display a reasonable gratitude to the Gods and eminent men by hymns.

His disciples were forbidden to eat beans, because, as they were flatulent, they greatly partook of animal properties; (that their stomachs would be kept in much better order by avoiding them), and that such abstinence would make the visions that appear in one's sleep gentle and free from agitation.

Alexander, in his Successions of Philosophers, reports the following doctrines as contained in Pythagoras's Commentaries: the Monad is the beginning of everything. From this proceeds an indefinite duad, which is subordinate to the monad, as to its cause. From the monad and the indefinite duad proceed numbers. From numbers proceed signs. From these, lines, of which plane figures consist. From these plane figures are derived solid bodies. From solid bodies are derived sensible bodies, of which last there are four elements, fire, water, earth and air. The

world, which is endued with life and intellect, and which is of a spherical figure, in its centre containing the earth, which is also spherical, and inhabited all over, results from a combination of these elements, and from them derives its motion. There are antipodes, and what to us is below, is to them above, He also taught that light and darkness, cold and heat, dryness and moisture, were equally divided in the world; and that, while heat was predominant in summer, so when cold prevailed, it was winter; when dryness prevailed, it was spring; and when moisture preponderated, autumn. The loveliest season of the year was when all these qualities were equally balanced; of which the flourishing spring was the most wholesome, and the autumn, the most pernicious. Of day, the most flourishing period was the morn while the evening was the fading one, and the least healthy.

Another of his theories was that the air around the earth was immovable, and pregnant disease, and that in it everything was mortal while the upper air was in perpetual motion, and salubrious; and that in it everything was immortal, and on that account divine. The sun, moon and the stars were all Gods; for in them dominates the principle which is the cause of. The moon derives its light from the sun. There a relationship between men and the Gods, because men partake of the divine principle; on which count, therefore, God exercises his providence for our advantage. Fate is the cause of the arrangement of the world, both in general and in particular. From the sun a ray penetrates both the cold aether, which is the air, *aer* and the dense aether, *pachun aithera*, which is the sea and moisture. This ray descends into the depths and in this way vivifies everything. Everything which partakes of the principle of heat lives, which account, also, plants are animated beings but that not all living beings necessarily have souls. The soul is something torn off from the aether, both warm and cold, from its partaking of the cold aether. The soul is something different from life. It is immortal, because of the immortality of that from which it was torn off.

Animals are born from one another by seeds and that it is impossible for there to be any spontaneous production by the earth. Seed is a drop from the brain which in itself contains a warm vapor; and that when this is applied to the womb, it transmits moisture, virtue, and blood from the brain, from which flesh, sinews, bones and hair, and the whole body are produced. From the vapor is produced the soul and also sensation. The infant first becomes a solid body at the end of forty days; but, according to the principles of harmony, it is not perfect till seven, or perhaps nine; or at most ten months, and then it is brought forth. In itself it contains all the principles of life which are all connected together, and by their union and combination form a hormonious whole, each of them developing itself at the appointed time.

In general the senses, and especially sights, are a vapor of intense heat, on which account a man is said to see through air, or through water. For the hot principle is opposed by the cold one; since, if the vapor in the eyes were cold, it would have the same temperature as the air, and so would be dissipated. As it is, in some passages he calls the eyes the gates of the sun. In a similar manner he speaks of hearing, and of the other senses. He also says that the soul of man is divided into three parts; into intuition (nous), reason (phren), and mind (thumos); and that the first and last divisions are found also in other animals, but that the middle one, reason, is found in man only. The chief abode of the soul is in those parts of the body which are between the heart and the brain. The mind abides in the heart, while the intuition (or deliberation) and reason reside in the brain.

The senses are drops from them; and the reasoning sense is immortal, while the others are mortal. The soul is nourished by the blood, and reasons are the winds of the soul. The soul is invisible, and so are its reasons, inasmuch as the aether itself is invisible. The links of the soul are the arteries, veins and nerves. When the soul is vigorous, and is by itself in a quiescent state, then its links are words and actions. When it is cast forth upon the earth, it wanders about, resembling the body. Mercury is the steward of the souls, and that is the reason of his name Conductor, Commercial, and Infernal, since it is he who conducts the souls from their bodies, and from earth, and sea; and that he conducts the pure souls to the highest region, and that he does not allow the impure ones to approach them nor to come near one another; committing them to be. bound in indissoluble fetters by the Furies. The Pythagoreans also assert that the whole air is full of souls, and that these are those that Page 169 DIOGENES LAERTES BIOGRAPHY XIX are accounted geniuses or heroes. They are the ones that send down among men dreams, and tokens of disease and health; the latter not being reserved to human beings, but being sent also to sheep and other cattle. They are concerned with purifications, expiations, and all kinds of divinations, oracular predictions, and the l(ike).

Man's most important privilege is to be able to persuade his soul to be either good or bad. (Men) are happy when they have a good soul; yet they never quiet, never long retaining the same mind. An oath is justice; and on that account Jupiter is called Jupiter of Oaths. Virtue is harmony, health, universal good and God; on which account everything owes its existence and preservation to harmony. Friendship is a harmonious quality. Honors to Gods and heroes should not be equal. The Gods should be honored at all times, extolling them with praises, clothed in white garments, and keeping one's body chaste; but that to the heroes such honors should not be payed till after noon.

A state of purity is brought about by purifications, washings and sprinklings; by a man's purifying himself from all funerals, concubinage, or any kind of pollution; by abstaining from all flesh that has either been killed or died of itself, from mullets, from melanuri, from eggs, from such animals as lay eggs, from beans, and from other things that are prohibited by those who have chared of the mysteries in the temples.

In his treatise on Beans, Aristotle says that Pythagoras's reason for demanding abstention from them on the part of his disciples, was that either they resemble parts of the human body, or because they are like the gates of hell Ñ they are the only plants without parts; -- or because they dry up other plants, or because they are representatives of universal nature, or because they are used in elections in oligarchical governments. He also forbade his disciples to pick up what fell from the table, for the sake of accustoming them to eat moderately, or else because such things belong to the dead. Aristophanes, indeed said that what fell belonged to the heroes, in his heroes singing, "Never taste the things which fall, From the table on the floor."

He also forbade his disciples to eat white poultry, because a cock of that color was sacred to the god Month, and was also a suppliant. He was also accounted a good animal (?) and he was sacred to the god Month, for he indicates the time.

The Pythagoreans were also forbidden to eat of all fish that was sacred, on the ground that the same animals should not be served up before both gods and men, just as the same things do not

belong to both freemen and slaves. Now white is an indication of a good nature, and black of a bad one.

Another of the precepts of Pythagoras was never to break bread; because in ancient, times friends used to gather around the same loaf, as they even now do among the barbarians. Nor would he allow men to divide bread which unties them. Some think that he laid down this rule in reference to the judgment which takes place in hell; some because this practice engenders timidity in war. According to others, the refence is to the Union, which presides over the government of the Universe.

Another one of his doctrines was that of all solid figures the sphere was the most beautiful; and of all plane figures, the circle. That old age, and all diminution was similar, and also; all increase and youth. That health was the permanence of form, and disease, its destruction. He thought salt should be set before people as a reminder of justice; for salt preserves everything which it touches, and is composed of the purest particles of water and the sea.

These are the doctrines which Alexander asserts that he discovered in th

ON SANCTITY

It is necessary that the laws should not be enclosed in houses, or by gates, but in the manners of the citizens. Which, therefore is the basic principle of any state? The education of the youth. For vines will never bear useful fruit, unless they are well cultivated; nor will horses ever excel, unless the colts are properly trained. Recently ripened fruit grows similar to its surroundings. With utmost prudence do men study how to prune and tend the vines; but to things pertaining to the education of their species the behave rashly and negligently; though neither wines nor wine govern men, but man and the soul of man. The nurture of a plant, indeed, we commit to an expert, who is supposed to deserve no less than two minae (a day); but the education of youth we commit to some Illyrian or Thracian, who is worthless. As the earliest legislators could not render the bourgeoisie stable, they prescribed (in the curriculum) dancing and rhythm, which instills motion and order; and besides these they added sports, some of which induced fellowship, but others truth and mental keenness. For those who thought that intoxication or guzzling had committed any crime, the prescribed the pipe of harmony, which by maturing and refining the manners so-shaped the mind that it became capable of culture.

It is well to invoke God at the beginning and end both of supper and dinner, not because he is in want of anything of the kind, but in order that the soul may be transfigured by the recollection of divinity. For since we proceed from him, and participate in a divine nature, we should honor him. Since also God is just, we also should act justly in all things.

In the next place, there are four causes which terminate all things; and bring them to an end; namely nature, law, art and fortune. Nature is admittedly the principle of all things. Law is the inspective guardian and creator of all things that change manners into political concord. Art is justly said to be the mother and guide of things consummated through human prudence. But of things which accidentally happen to the worthy and unworthy, the cause is ascribed to fortune, which does not produce anything orderly, moderate, or controlled.

THEAGES

ON THE VIRTUES

The soul is divided into reasoning power, anger and desire. Reasoning power rules knowledge, anger deals with impulse, and desire bravely rules the soul's affections. When these three parts unite into one action, exhibiting a composite energy, then in the soul results concord and virtue. When sedition divides them, then appear discord and vice. Virtue therefore contains three elements; reason, power, and deliberate choice. The soul's reasoning power's virtue is prudence, which is a habit of contemplating and judging. The irascible part's virtue is fortitude; which is a habit of enduring dreadful things, and resisting them. The appetitive part's virtue is temperance; which is a moderation and detention of the pleasures which arise from the body. The whole soul's virtue is justice; for men indeed become bad either through vice, or through incontinence, or through a natural ferocity. They injure each other either through gain, pleasure or ambition. More appropriately therefore does vice belong to the soul's reasoning part. While prudence is similar to good art, vice resembles bad art, inventing contrivances to act unjustly. Incontinence pertains to the soul's appetitive part, as continence consists in subduing, and incontinence in failure to subdue pleasures. Ferocity belongs to the soul's irascible part, for when some ennactivated by evil desires is gratified not as a man should be, but as a beast would be, then this is called ferocity.

The effects of these dispositions also result from the things for the sake of which they are performed. Vice, hailing from the soul's reasoning part results in avarice; the irascible part's fault is ambition, which results in ferocity; and as the appetitive part ends in pleasure, this generates incontinence. As unjust actions are the results of so many causes, so also are just deeds; for virtue is a naturally beneficent and profitable as vice is maleficent and harmful.

Since, however, of the parts of the soul one leads while the others follow, and since the virtues and vice subsist about these and in these, it is evident that with respect to the virtues also, some are leaders and others followers, while others are compounds of these. The leaders are such as prudence; the followers being fortitude and temperance; their composites are such as justice. How the virtues subsist in and about the passions, so we may call the latter the matter of the former. Of the passions, one is voluntary, and the others involuntary; pleasure being the voluntary, and pain the involuntary. Men who have the political virtues increase and decrease these, organizing the other parts of the soul to that which possesses reason. The desirable point of this adaptation is that intellect should not be prevented from accomplishing its proper work, either by lack of excess. We adapt the less good to that which is more so; as in the world every part that is always passive subsists for the sake of that which is always moved. In the conjunction of animals, the female subsists for the sake of the male; for the latter sows, generating a soul, while the former alone imparts matter to that which is generated. In the soul, the irrational subsists for the sake of the rational part. Anger and desire are organized in dependence on the first part of the soul, the former as a satellite and guardian of the body, the latter as a dispenser and provider of necessary wants. Intellect being established in the highest summit of the body, and having a prospect in that which is on all sides splendid and transparent, investigates the wisdom of real beings. This indeed is its natural function, to investigate and obtain possession of the truth, and to follow those beings which are more excellent and honorable than itself. For the

knowledge of things divine and most honorable is the principle, cause and rule of human blessedness.

The principles of all virtue are three; knowledge, power and deliberate choice. Knowledge indeed is that by which we contemplate and form a judgment of things; power is a certain strength of nature from which we derive our subsistence, and which gives stability to our actions; and deliberate choice is as it were the hands of the soul by which we are impelled to, and lay hold on the objects of our choice...When the reasoning power prevails over the irrational part of the soul, then endurance and continence are produced; endurance indeed in the retention of pains, but continence in the absence of pleasures. But when the irrational parts of the soul prevail over the reasoning part of the soul, then are produced effeminacy in flying from pain, and incontinence in being vanquished by the pleasures. When however the better part of the soul prevails, the less excellent part is governed; the former leads, and the latter follows, and both consent and agree, and then in the whole soul is generated virtue and all the goods. Again, when the appetitive part of the soul follows the reasoning, then is produced temperance, when this is the case with the irascible, appears fortitude; and when it takes place in all the parts of the soul, then the result is justice. Justice is that which separates all the vices and all the virtues of the soul from each other. Justice is an established order and organization of the parts of the soul, and the perfect and supreme virtue; in this every good is contained, while the other goods of the soul cannot subsist without it. Hence justice possesses great influence both among gods and men. It contains the bond by which the whole and the universe are held together, and also that by which the gods and men are connected. Among the celestials it is called Themis, and among the terrestrials it is called Dice; while among men it is called the Law. These are but symbols indicative that justice is the supreme virtue. Virtue, therefore, when it consists in contemplating and judging, is called prudence; when in sustaining dreadful things, is called fortitude; when in restraining pleasure, it is called temperance; and when in abstaining from injuring our neighbors, justice.

Obedience to virtue according to, and transgression thereof contrary to right reason, tend towards decorousness, and its opposite. Propriety is that which ought to be. This requires neither addition or detraction, being what it should be. The improper is of two kinds: excess and defect. The excess is over -scrupulousness, and its deficiency, laxity. Virtue however is a habit of propriety. Hence it is both a climax and a medium of which are proper things. They are media because they fall between excess and deficiency; they are climaxes, because they endure neither increase nor decrease, being just what they ought to be.

Since however the virtue of manners consists in dealing with the passions, over which pleasure and pain are supreme, virtue evidently does not consist in extirpating the passions of the soul, pleasure and pain, but in regulating them. Not any more does health, which is an adjustment of the bodily powers, consist in expelling the cold and the hot, the moist and the dry, but in adjusting them suitably and symmetrically. Likewise in music, concord does not consist in expelling the sharp and the flat, but in exterminating dissonance by concord arising from their adjustment. Therefore it is the harmonious adjustment of heat and cold, moisture and dryness which produces health, and destroys disease. Thus by the mutual adjustment of anger and desire, the vices and other passions are extirpated, while virtue and good manners are induced. How the greatest peculiarity of the virtue of manners in beauty of conduct is deliberate choice. Reason

and power may be used without virtue, but deliberate choice cannot be used without it; for deliberate choice inspires dignity of manners.

When the reasoning power by force subdues anger and desire, it produces continence and endurance. Again when the reasoning force is dethroned violently by the irrational parts, then result incontinence and effeminacy. Such dispositions of the soul as these are half-perfect virtues and vices. For (according to its nature) the reasoning power of the soul induces health, while the irrational induces disease. So far as anger and desire are governed and led by the soul's rational part, continence and endurance become virtues; but in so far as this is effected by violence, involuntarily, they become vices. For virtue must carry out what is proper not with pain but pleasure. So far as anger and desire rule the reasoning power there is produced effeminacy and incontinence, which are vices; but in so far as they gratify the passions with pain, knowing that they are erroneous in consequence of the eye of the soul being healthy, so far as this is the case, they are not vices. Hence it is evident that virtue must voluntarily do what is proper, as the involuntary implies pain and fear, while the voluntary implies pleasure and delight.

This may be corroborated by division. Knowledge and the perception of things are the province of the rational part of the soul; while power pertained to the irrational part, whose peculiarity is in an ability to resist pain, or to vanquish pleasure. Both of these, the rational and the irrational, subsists deliberate choice, which consists of intention and appetite, intention pertaining to the rational part, and appetite to the irrational. Hence every virtue consists in a mutual adaptation of the souls parts while, both will and deliberate choice subsist entirely in virtue. In general, therefore, virtue is a mutual [adaptation] of the irrational part of the soul to the irrational to the irrational. Virtue, however, is produced through pleasure and pain striking the right resultance of propriety. But propriety is that which ought to be, and the improper, what ought not;..... The fit and the unfit are to each other as the equal and the unequal, as the ordered and the disordered; of, which the two former are finite, and the two latter are the infinite (limit and infinity are the two great principles of things, below the universal ineffable cause). On this account the parts of the unequal are referred to the middle, but not to each other. An angle greater than a right angle is called obtuse; the acute one being less than it. (In a circle) also, the right line is greater than the radius, drawn from the centre. Any day beyond the equinox is greater is greater than it. Overheat or undercold produce diseases. Overheatedness exceeds moderation, which over-coldness does not reach.

The same analogy holds good in connection with the soul. Boldness is an excess of propriety in the endurance of things of a dreadful nature; while, timidity is a deficiency. Prodigality is an excess of proper expenditure of money; while illiberality is its excess. Rage is an excess of the proper use of the soul's irascible part, while insensibility ii the corresponding deficiency. The same reasoning applies to the opposition of the other dispositions of the soul. Since however virtue is a habit of propriety, and a medium of the passions, it should be neither wholly passive, nor immoderately passive. Impassivity causes unimpelledness of the soul and lack of enthusiasm for the beautiful in conduct, while immoderate passivity perturbs the soul, and makes it inconsiderate. We should then, in virtue, see passions as shadow and outline in a picture; which depend on animation and delicacy, imitation of the truth and contrast of coloring. The soul's passions are animated by the natural incitation and enthusiasm of virtue, which is generated from

the passions, and subsisting with them. Similarly, harmony includes the sharp and the flat, and mixtures consist of heat and cold, and equilibrium results from weight and lightness.

Therefore, neither would it be necessary nor profitable to remove the passions of the soul: but they must be mutually adjusted to the rational part; under the direction of propriety and moderation.

METOPUS

CONCERNING VIRTUE

Man's virtue is the perfection of his nature. By the proper nature of his virtue, every being becomes perfect, and arrives at the climax of its excellence. Thus the virtue of the horse is that which makes the best of the horse's nature. The same reasoning applies to details. Thus the virtue of the eyes is acuteness of vision; and this is the climax of the eyes' nature. The virtue of the ears is acuteness of hearing; and this is the [aural] nature's climax. The virtue of the feet is swiftness; and this is the pedal nature's climax.

Every virtue, however, should include these three things: reason, power, and deliberate choice. Reason indeed, judges and contemplates; power prohibits and vanquishes; and deliberate choice loves and enjoys propriety. Therefore to judge and contemplate pertain to the intellectual part of the soul; to prohibit and vanquish are the peculiarity of the irrational part of the soul; and to love and enjoy propriety includes both rational and irrational parts of the soul; for deliberate choice consists of the discursive energy of reason, and appetite. Intention therefore, pertains to the rational, but appetite to the irrational parts of the soul.

We may discern the multitude of the virtues by observing the parts of the soul; also the growth and nature of virtue. Of the soul's parts, two rank first: the rational and the irrational. It is by their rational that we judge and contemplate; by the irrational we are impelled and desire. These are either consonant or discordant, their strife and dissonance being produced be excess or defect. The rational part's victory over the irrational produces endurance and continence. When the rational leads, the irrational follows, both accord, and produce virtue. That is why endurance and continence are generally accompanied by pain; for endurance resists pain and continence pleasure. However, incontinence and effeminacy neither' resist nor vanquish pleasure. That is why men fly from good through pain, but reject it through pleasure. Likewise praise and blame, and everything beautiful in human conduct, are produced in these parts of the soul. This explains the nature, of virtue.

Let us study virtue's kinds and parts. Since the soul is divided into two parts, the rational and the irrational, the latter is also divided into two, the irascible and appetitive part. By the rational we judge and contemplate; by the irrational we are impelled and desire. The irascible part defends us, and revenges incidental molestations; the appetitive directs and preserves the body's proper constitution. So we see that the numerous virtues with all their differences and peculiarities do little more than conform to the distinctive parts of the soul.

TESTS OF PYTHAGOREAN INITIATION

As he therefore thus prepared his disciples for culture, he did not immediately receive as an associate any who came to him for that purpose until he had tested them and examined them judiciously. To begin with he inquired about their relation to their parents and kinsfolk. Next he surveyed their laughter, speech or silence, as to whether it was unreasonable; further, about their desires, their associates, their conversation, how they employed their leisure, and what were the subjects of their joy or grief. He observed their form, their gait, and the whole motions of their body. He considered their frame's natural indications physiognomically, rating them as visible exponents of the invisible tendencies of the soul. After subjecting a candidate to such trials, he allowed him to be neglected for three years, still covertly observing his disposition towards stability, and genuine studiousness, and whether he was sufficiently averse to glory, and ready to despise popular honors.

After, this the candidate was compelled to observe silence for five years, so as to have made definite experiments in continence of speech, inasmuch as the subjugation of the tongue is the most difficult of all victories, as has indeed been unfolded by those who have instituted the mysteries. During this probation, however, the property of each was disposed of in common, being committed to trustees, who were called politicians, economizers or legislators. Of these probationers, after the quinquennial silence, those who by modest dignity had won his approval as worthy to share in his doctrines, then became *esoterics*, and within the veil both heard and saw Pythagoras. Prior to this they participated in his words through the hearing alone, without seeing him who remained within the veil, and themselves offering to him a specimen of their manners. If rejected, they were given the double of the wealth they had brought, but the *auditors* raised to him a tomb, as if they were dead; the disciples being generally called *auditors*.

Should these later happen to meet the rejected candidate, they would treat him as a stronger, declaring that he whom they had by education modeled had died, inasmuch as the object of these disciplines had been to be turned out good and honest men.

Those who were slow in the acquisition of knowledge were considered to be badly organized or, we may say, deficient, and sterile.

If, however, after Pythagoras had studied them physiognomically, their gait, motions and state of health, he conceived good hopes of them; and if, after the five years' silence, and the emotions and initiations from so many disciplines together with the ablutions of the soul, and so many and so great purifications produced by such various theorems, through which sagacity and sanctity is ingrained into the soul.....if, after all this even, someone was found to be still sluggish and dull, they would raise to such a candidate within the school a pillar or monument, such as was said to have been done to Perialus the Thurian, and Cylon the prince of the Sybarites, who were rejected, they expelled him from the auditorium, loading him down with silver and gold. This wealth had by them been deposited in common, in the care of certain custodians, aptly called *Economics*. Should any of the Pythagoreans later meet with the reject, they did not recognize him whom they accounted dead. Hence also Lysis, blaming a certain Hipparchus for having revealed the Pythagorean doctrines to the profane, and to such as accepted them without disciplines or theory, said:

"It is reported that you philosophise indiscriminately and publicly, which is opposed to the customs of Pythagoras. With assiduity you did indeed learn them, O Hipparchus; but you have not preserved them. My dear fellow, you have tasted Sicilian tit-bits, which you should not have repeated. If you give them up, I shall be delighted; but if you do not, you will to me be dead. For it would be pious to recall the human and divine precepts of Pythagoras, and not to communicate the treasures of wisdom to those who have not purified their souls, even in a dream. It is unlawful to give away things obtained with labors so great, and with assiduity so diligent to the first person you meet, quite as much as to divulge the mysteries of the Eleusynian goddesses to the profane. Either thing would be unjust and impious. We should consider how long a time was needed to efface the stains that had insinuated themselves in our breasts, before we became worthy to receive the doctrines of Pythagoras. Unless the dyers previously purified the garments in which they wish the desired colors to be fixed, the dye would either fade, or be washed away entirely. Similarly, that divine man prepared the souls of lovers of philosophy, so that they might not disappoint him in any of those beautiful qualities which he hoped they would possess. He did not impart spurious doctrines, nor stratagems, in which most of the Sophists, who are at leisure for no good purpose, entangle young men; but his knowledge of things human and divine was scientific. These Sophists, however, use his doctrines as a mere pretext commit dreadful atrocities, sweeping the youths away as in a dragnet, most disgracefully, making their auditors become rash nuisances. They infuse theorems and divine doctrines into hearts whose manners are confused and agitated, just as if pure, clear water should be poured into a deep well full of mud, which would stir up the sediment and destroy the clearness of the water. Such a mutual misfortune occurs between such teachers and disciples. The intellect and heart of those whose initiation has not proceeded by disciplines, are surrounded lay thickets dense and thorny, which obscure the mild, tranquil and reasoning power of the soul, and impede the development and elevation of the intellective part. These thickets are produced by intemperance and avarice, both of which are prolific. Intemperance produces lawless marriages, lusts, intoxications, unnatural enjoyments, and passionate impulsions which drive headlong into pits and abysses. The unbridling of desires has removed the barriers against incest with even mothers or daughters, am just as a tyrant would violate city regulations, or country's laws, with their hands bound behind them, like slaves, they have been dragged to the depths of degradation. On the other hand, avarice produces rapine, robbery, parricide, sacrilege, sorcery, and kindred evils. Such being the case, these surrounding thickets, infested with passions, will have to be cleared out with systematic disciplines, as if with fire and sword; and when the reason will have been liberated from so many and great evils, we are in a position to offer to it, and implant within it something useful and good."

So great and necessary was the attention which, according to Pythagoras, should be paid to disciplines as introductions to philosophy.

Moreover, inasmuch as he devoted so much care to the examination of the mental attitudes of prospective disciples, he insisted that the teaching and communication of his doctrines should be distinguished by great honor.

USE OF PARABLES OF INSTRUCTION

Pythagoras considered most necessary the use of parables in instruction. Most of the Greeks had adopted it, as the most ancient; and it had been both preferentially and in principle employed by the Egyptians, who had developed it in the most varied manner. In harmony with this it will be found that Pythagoras attended to it sedulously, if from the Pythagoric symbols we unfold their significance and arcane intentions, developing their content of rectitude and truth, liberating them from their enigmatic form. When, according to straightforward and uniform tradition they are accommodated to the sublime intelligence of these philosophers, they deify beyond human conception.

Those who came from this school, not only the most ancient Pythagoreans, but also those who during his old age were still young, such as Philolaos, and Eurytus, Charendas and Zaleucus, Brysson and the elder Archytas, Aristaeus, Lysis and Empdocles, Zamoixis and Epimanides, Milo and Leucippus, Alcmaeon and Hippasus, and Thymaridas were all of that age, a multitude of savants, incomparably excellent, --- all these adopted this mode of teaching, both in their conversations, and commentaries and annotations. Their writings also, and all the books which they published, most of which have been preserved, to our times, were not composed in popular or vulgar diction, or in a manner usual to all other writers, so as to be immediately understood, but in a way such as to be not easily apprehended by their readers. For they adopted Pythagoras's law of reserve, in an arcane manner concealing divine mysteries from the uninitiated, obscuring their writings and mutual conversations.

The result is that they who presents theses symbols without unfolding their meaning by a unsuitable exposition, runs the danger of exposing them to the charge of being ridiculous and inane, trifling and garrulous. When however they expounded according to these symbols, and made clear and obvious even to the crowds, then they will be found analogous to prophetic sayings such as the oracles of the Pythian Apollo. Their admirable meaning will inspire those who unite intellect and scholarliness.

It might be well to mention a few of them, explain this mode of discipline.

Not negligently enter into a temple or adore carelessly, even if only at the doors.
Sacrifice and adore unshod.
Shunning public roads, walk in unfrequented paths.
Not without light speak about Pythagoric affairs.

Such is a sketch of the symbolic mode of teaching adopted by Pythagoras.

TIMAEUS LOCRIUS, The Teacher of Plato, on THE SOUL AND THE WORLD

MIND, NECESSITY, FORM & MATTER

Timaeus the Locrian asserted this: Ñ that of all the things in the Universe, there are, two causes, (one) Mind, (the cause) of things existing according to reason; (the other) Necessity, (the cause) of things (existing) by (some) force, according to the power of the bodies; and that the former of these is the nature of the good and is called God, and the principle of things that are best; but what accessory causes follow, are referred to Necessity. As regards the things in the Universe, there are Form, Matter, and the perceptible; which is, as it were, a resistance of the two others; and that Form is unproduced, and unmoved, and stationary and of the nature of the same, and perceptible by the mind, and a pattern of such things produced, as exist by a state of change; for that some such thing as this is Form, spoken of and conceived to be.

Matter, however, is a mold, and a mother and a nurse, and procreative of the third kind of being; for receiving upon itself the resemblances, and as it were remolding them, it perfects these productions. He asserted moreover that Matter, though eternal is not unmoved; and though of itself it is formless and shapeless, yet it receives every kind of form; and that what is around bodies, is divisible, and partakes of the nature of the different; and that Matter is called by the twin names of Plane and Space. These two principles, then, are opposite to each, other; of which Form relates to a male power, and a father; while matter relates to a female, and a mother. Being three, they are recognisable by three marks: Form, by mind, according to knowledge; Matter by a spurious kind of reasoning, because of its not being mentally perceived directly, but by analogy and their productions by sensation and opinion.

CREATION OF THE WORLD

Before the heavens, then, there existed through reason, Form and Matter, and the God who develops the best. But since the older surpasses the younger and the ordered surpasses the orderless, the deity being good, on seeing that Matter receives Form, and is altered in every way, but without order, the necessity of organizing it, altering the undefined to the defined, so that the differences between bodies might be similarly related, not receiving various turns at hap-hazard. He therefore made this world out of the whole of Matter, laying it down as a limit to the nature of being, through its containing in itself all the rest of things, being one, only-begotten, perfect, endued with soul and reason, for these qualities are superior to the soul-lees and the irrational, Ñ and of a sphere like body; for this is more perfect than the rest of forms.

Desirous then of making a very good production, he made it a deity, created and never to be destroyed by any cause other than the God, who had put it in order, if indeed he should ever wish to dissolve it. But on the part of the good there is no rushing forward to the destruction of a very beautiful production. Such therefore being the world, it continues without corruption and destruction, being blessed. It is the best of things ordered; since it has been produced by the best cause, that looks not to patterns made by hand, but to Form in the abstract, and to Existence, perceived by the mind to which the created thing, having been carefully adjusted, has become the most beautiful, and to be not wrongly undertaken. It is [ever]perfect according to the things

perceived by sense because the pattern perceived by mind contains [in] itself all the living things perceived by mind; he left out of itself nothing, as being the limit [of]the things perceived by mind, as this world is [of] those perceived by sense.

As being solid, and perceptible by touch and sight, it has a share of earth and fire, and of the things between them, air and water; and it is composed of bodies all perfect, which are in it as wholes so that no part might ever be left out of it, in order that the body of the Universe might be altogether self-sufficient, uninjured by corruption without or within; for apart from these there is nothing else, for the things combined according the best proportions and with equal powers, neither rule over, nor are ruled by each other in turn, so that some receive an increase, others a decrease, remaining indissolubly united according to the very best proportions.

PROPORTIONS OF THE WORLD-COMBINATION

For whenever there are any three terms, with mutually equal intervals, that are proportionate, we then perceive that, after the manner of an extended string, the middle is to the first, as is the third to it; and this holds true inversely and alternately, interchanging places and order; so that it is impossible to arrange them numerically without producing an equivalence of results. Likewise the world's shape and movement are well arranged; the shape is a sphere self similar on all sides, able to contain all shapes that are similar; the movement endlessly exhibits the change dependent on a circle.

Now as the sphere is on every side equidistant from the centre, it is able to retain its poise whether in movement or at rest; neither leaving its poise, nor assuming another. Its external appearance being exactly smooth, it needs no mortal organs such as are fitted to, and present in all other living beings, because of their wants. The world-soul's element of divinity radiates out from the centre, entirely penetrating the whole world, forming a single mixture of divided substance with undivided form; and this mixture of two forces, the same and the different, became the origin of motion; which indeed was not accomplished in the easiest way, being extremely difficult.

Now all these proportions are combined harmonically according to numbers; which proportions were scientifically divided according to scale which reveals the elements and the means of the soul's combination. Now seeing that the earlier is more powerful in power and time than the later, the deity did not rank the soul after the substance of the body, but made it older, by taking the first of unities, 384 (12 x 16)[x 2?]. Knowing this first, we can easily reckon the double and the triple and all the terms together, with the complements and eighths, must amount to 114,69 and likewise the divisions (sum of the tone sequences of 36 tones, amounting to 384 x 27, the perfect cube).

PLANETARY REVOLUTIONS AND TIME

God the eternal, the chief ruler of the Universe, and its creator is beheld alone by the mind; but we may behold by sight all that is produced this world and its parts, how many soever they [----] in heaven; which as being ethereal, must be [divided] into kinds, some relating to sameness, others to difference. Sameness draws inward all that is without, along the general eastward

movement from the West. Difference draws from within all self-moved portions from West to East, fortuitously rolling around and along by the superior power of sameness.

The different's movement being divided in harmonical proportion, assumes the order of [seven] circles Nearest to the earth, ,the Moon revolves in a month; while beyond her the Sun completes his revolution in a year. Two planets run a [co---]equal with that of the Sun: Mercury, and Juno, also called Venus and Lucifer, because shepherds and people generally are not skillful in sacred astronomy, confusing the western and eastern rise. The same star may shine in the West when following the Sun at a distance great enough to be visible in spite of solar splendor; and at another time in the East, when, as herald of the day it rises before the Sun, leading it. Because of its [running] together with the sun, Venus is Lucifer frequently but not always; for there are planet and stars of any magnitude seen above the horizon before sunrise, herald the day. But the three other planets, Mars, Jupiter and Saturn have their peculiar velocities and different years, completing their course while making their periods of effulgence, of visibility, of obscuration and eclipse, causing accurate rising and settings. Moreover they complete their appearances conspicuously in East or West according to their position relative to the Sun, who during the day speeds westward, which during the night it reverses, under the influence of sameness; while its annual revolution is due to its inherent motion. In resultance of these two kinds of motion it rolls out a spiral, creeping according to one portion, in the time of a day, but, whirled around under the sphere of the fixed stars, according to each revolution of darkness and day.

Now these revolutions are by men called portions of time, which the deity arranged together with the world. For before the world the stars did not exist; and hence there was neither year, nor periods of seasons, by which this generated time is measured, and which is the representation of the ungenerated time called eternity. For as this heaven has been produced according to an eternal pattern, (the world of ideas), Ñ so according to the pattern of eternity was our world-time created simultaneously with the world.

THE EARTH'S CREATION BY GEOMETRIC FIGURES

The Earth, fixed at the centre, becomes the hearth of the gods, and the boundary of darkness and day, producing both settings and risings, according to the occultations produced by the things that form the boundary, just as we im prove our sight by making a tube with our closed hand, to exclude refraction. The Earth is the oldest body in the heavens. Water was not produced without Earth, nor air without moisture; nor could fire continue without moisture and the materials that are inflammable; so that the Earth is fixed upon its balance as the root and base of all other substances. Of produced things, the substratum is Matter, while the reason of each shape is abstract Form; of these two the resultance is Earth, and Water, Air and Fire.

This is how they were created. Every body is composed of surfaces, whose elements are triangles; of which one is right-angled, and the other has all unequal sides, with the greater angle thrice the size of the lesser; while its least angle is the third of a right angle, and the middle [one] is double of the least; for it is two parts out of three; while the greatest is a right angle, being one and a half greater than the middle one, and the triple of the least. Now this unequal [sided] triangle is the half of a equilateral triangle, out into two equal parts by a line let down from apex to that base. Now in each of these triangles there is a right angle; but in the one, the two sides

about the right angle are equal, and in the other, all the sides are unequal. Now let this be called a scalene triangle; while the other, the half of the square, is the principle of the constitution the Earth. For the square produced from this scalene triangle is composed of four half squares and from such a square is produced the cube, a [body] the most stationary and steady in every way; having six sides and eight angles, and on this account [the]Earth is a body the heaviest and most difficult [to] be moved and its substance is inconvertible because it has no affinity with a triangle of any kind. Only the Earth has as peculiar element [the--] square and this is the element of the three other substances, Fire, Air and Water. For when the half triangle is put together six times, it produces a solid equilateral triangle; the exemplar of the [pyramid], which has four faces with equal angles, which is the form of Fire, as the easiest to be moved and composed of the finest particles. After this ranks the octohedron, with eight faces and six angles, the element of Air, and the third is the icosahedron, with twenty faces and twelve angles, the element of Water, composed of the most numerous and heaviest particles.

These then, as being composed of the same element, are transmuted. But the deity has made the dodecahedron, as being the nearest to the sphere, the image of the Universe. Fire then, by the fineness of its particles, passes through all things; and Air through the rest of things, with the exception of Fire; and Water through the Earth. All things are therefore full, and have no vacuum. They cohere by the revolving movement of the Universe, and are pressed against, and rubbed by, each other in turn, and produce the never-failing change from production to destruction.

CONCRETION OF THE ELEMENTS

By making use of these the deity put together this world, sensible to touch through the particles of Earth, and to sight through those of Fire; which two are the extremes. Through the particles of Air and Water he has conjoined the world by the strongest chain, namely, proportion; which restrains not only itself but all its subjects. Now if the conjoined object is a plane surface, one middle term is sufficient; but if a solid, there will be need of two. With two middle terms, therefore, he combined two extremes; so that as Fire is to Air, Air might be to Water, and Water to Earth; and by alternation, as Fire is to Water, Air might be to Earth; and by inversion as Earth is to Water, Water might be to Air, and Air to Fire; and by alternation, as Earth is to Air, so Water might be to Fire. Now since all are equal in power, their ratios are in a state of equality. This world is then one, through the bond of the deity, made according to proportion.

How each of these substances possesses many forms; Fire, those of Flame, and Burning and Luminousness, through the inequality of the triangles in each of them. In the same manner, Air is partly clear and dry, and partly turbid and foggy; and Water partly flowing and partly congealed, according as it is Snow, Hoar-frost, Hail or Ice; and that which is Moist, is in one respect flowing as honey and oil; but in another is compact, as pitch and wax; and of compact-forms there are some fusible, as gold, silver, copper, tin, lead and steel; and some friable, as sulphur, pitch, nitre, salt, alum, and similar metals.

COMPOSITION OF THE SOUL

After putting together the world, the deity planned the creation of living beings, subject to death, so that, himself being perfect, he might perfectly work it out according to his image. Therefore he mixed up the soul of man out of the same proportions and powers, and after taking the particles and distributing them, he delivered them? over to Nature, whose office is to effect change. She then took up the task of working out mortal and ephemeral living beings, whose souls ware drawn in from different sources, some from the Moon, others from the Sun, and others from various planets, that cycle within the Difference, -- with the exception of one single power which was derived from Sameness, which she mixed up in the rational portion of the soul, as the image of wisdom in those of a happy fate.

Now of the soul of man one portion is rational and intellectual; and another irrational and unintellectual. Of the logical part, the best portion is derived from Sameness, while the worse comes from Difference; and each is situated around the head, so that the other portions of the soul and body may minister to it, as the uppermost of the whole tabernacle. Of the irrational portion, that which represents passion hangs around the heart, while desire inhabits the liver. The principle of the body, and root of the marrow is the brain, wherein inheres leadership; and from this, like an effusion, through the back-bone flows what remains, from which are separated the particles for seed and reason; while the marrow's surrounding defences are the bones, of which the flesh is the covering and concealment. To the nerves he united joints by ligatures, suitable for their movement. Of the internal organs, some exist for the sake of nourishment, and others for safety; of communications, some convey outside movements to the interior intelligent places of perception, while others, not falling under the power of apprehension, are unperceived, either because the affected bodies are too earthlike, or because the movements are too feeble; the painful movements tend to arouse Nature, while the pleasurable lull Nature into remaining within itself.

SENSATIONS

Amongst the senses, the deity has in us lit sight to view the objects in the heavens, and for the reception of knowledge; while to make us capable to receive speech and melody, he has in us implanted hearing, of which he who is deprived thereof from birth will become dumb, nor be able to utter any speech, and that is why this sense is said to be related closest to speech. .As many affections of the body as have a name are so called with reference to touch; and others from relation to their seat. Touch judges of the properties connected with life, such as warmth, coldness, dryness, moisture, smoothness, roughness, and of things, that they are yielding, opposing, hard, or soft. Touch also decides of heaviness or lightness. Reason defines these affections as being centripetal and centrifugal; which men mean to express when they say below, and middle. For the centre of a sphere is below, and that part lying above it and stretching to the circumference, is called upwardness.

Now what is warm appears to consist of fine particles, causing bodies to separate; while coldness consists of the grossness of the particles, causing a tendency to condense.

The circumstances connected with the sense of taste are similar to those of touch. For substances grow either smooth or rough by concretion and secretion, by entering the pores, and assuming shapes. For those that cause the tongue to melt away, or that scrape it, appear to be rough; while those that act moderately in scraping appear brackish; while those that inflame or separate the skin are acrid; while their opposites, the smooth sweet, are reduced to a juicy state.

Of smelling, the kinds have not been defined; for, because of their percolating through narrow pores, that are too stiff to be closed or separated, things seem to be sweet-smelling or bad-smelling from the putrefaction or concoction of the earth and similar substances.

A vocal sound is a percussion in the air, arriving at the soul through the ears; the pores (or communications) of which reach to the liver; and among them is breath, by the movement of which hearing exists. Now of the voice and hearing, that portion which is quick is acute; while that which is slow, is grave; the medium being the most harmonious. What is much and diffused, is great; what is little and compressed, is small; what is arranged according to musical proportions is in tune, while that which is unarranged, and unproportionate, is out of tune, and not properly adjusted.

The fourth kind of things relating to the senses is the most multiform and various, and they are called objects of sight, in which are all kinds of colors, and an infinity of colored substances. The principle are four: white, black, brilliant (blue) and red, out of a mixture of which all other colors are prepared. What is white causes the vision to expand, and what is black causes it to contract; just as warmth expands, and cold contracts, and what is rough contracts the tasting, and what is sharp dilates it.

RESPIRATION

It is natural for the covering of animals that live in the air to be nourished and kept together by the food being distributed by the veins through the whole mass, in the manner of a stream, conveyed as it were by channels, and moistened by the breath, which diffuses it, and carries it to the extremities. Respiration is produced through there being no vacuum in nature; while the air, as it flows in, is inhaled in place of that which is exhaled, through unseen pores such as those through which perspiration -drops appear on the skin; but a portion is excreted by the natural warmth of the body. Then it becomes necessary for an equivalent portion to be reintroduced, to avoid a vacuum, which is impossible, for the animal would no longer be concentrating, and single, when the covering had been separated by the vacuum.

Now in lifeless substances, according to the analogy of respiration, the same organization occurs. The gourd, and the amber, for instance, bear resemblance to respiration.

Now the breath flows through the body to an orifice outwards, and is in turn introduced through respiration by the mouth and nostrils, and again after the manner of the Euripus, is in turn carried to the body, which is expended according to the expiration. Also the gourd, when the air within is expelled by fire, attracts moisture to itself; and amber, when the air is separated from it, receives an equal substance. Now all nourishment comes as from a root from the heart; and from the stomach; as a fountain; and is conveyed to the body, to which, if it be moistened by more

than what flows out, there is said to be an increase; but if less, by a decay; but the point of perfection is the boundary between these two, and is considered to exist in an equality of efflux and influx; but when the joints of the system are broken, should there no longer exist any passage for the breath, or the nourishment not be distributed, then the animal dies.

DISORDERS

There are many things hurtful to life, which are causes of death. One kind is disease. Its beginning is disharmony of the functions, when the simple powers, such as heat, cold, moisture or dryness are excessive or deficient. Then come turns and alterations in the blood, from corruption, and the deterioration of the flesh, when wasting away, should the turns take place according to the changes, to what is acid, or brackish, or bitter, in the blood, or wasting away of the flesh. Hence arise the production of bile, and of phlegm, diseased juices, and the rottenness of liquids weak indeed, unless deeply seated; but difficult to cure, when their commencement is generated from the bones, and painful, if in a state of inflammation of the marrow. The last of disorders are those of the breath, bile and phlegm, when they increase and flow into situations foreign to them, or into places inappropriate for them, by laying hold, of the situation, belonging to what is better, and be driving away what is congenial they fix themselves there, injuring the bodies, and resolving them into the very things.

These then are the sufferings of the body; and hence arise many diseases of the soul; some from one faculty, and some from another. Of the perceptive soul the disease is a difficulty of perception, of the recollecting, a forgetfulness of the appetitive part, a deficiency of desire and eagerness; of the affective, a violent suffering and excited madness; of the rational, an indisposition to learn and think.

But of wickedness the beginnings are pleasure and pains; desires and fears, inflamed by the body, mingled with the wind and called by different names. For there loves and regrets, desires let loose, and passions on the stretch, heavy resentments, and appetites of various kinds, and pleasures immoderate. Plainly, to be unreasonably disposed towards the affections is the limit of virtue, and to be under their rule is that of vice; for to abound in them, or to be superior to them, places us in a good or bad position. Against such impulses the temperaments of our bodies is greatly able to cooperate, whether quick or hot, or various, by leading us to melancholy or violent lewdness; and certain parts, when affected by a catarrh, produce itchings and forms of body more similar to a state of inflammation than one of health; through which a sinking of the spirits and a forgetfulness, a stillness and a state of fear are witnessed.

DISCIPLINE

Important, too, are the habits in which persons are trained, in the city or at home, and their daily food, by luxury enervating the soul, or fortifying it for strength. For the living out of doors, and single fare, and gymnastic exercises, and the morals of companions, produce the greatest effect in the way of vice and virtue. These causes are derived from our parents and the elements, rather than ourselves, provided that on our part there be no remissness, by keeping aloof from acts of duty. The animal cannot be in good condition unless the body possesses the better properties under its control; namely, health and correct perception, and strength and beauty.

Now the principles of beauty are a symmetry as regards its parts, and as regards the soul. For nature has arranged the body, like an instrument to be subservient to, and in harmony with, the subjects of life. The soul must likewise be brought into harmony with its analogous good qualities, namely, in the case of temperance, as the body is in the case of health; and in that of prudence, as in the case of correct perception; and in that of fortitude, as in the case of vigor and strength; and in that of justice, as in the case of beauty.

Nature, of course, furnishes their beginnings; but their continuation and maturation result from carefulness; those relating to the body, through the gymnastic and medical arts, those to the soul through instruction and philosophy. For these are the powers that nourish and give a tone to the body and soul by means of labor and gymnastic exercise, and pureness of diet; some through drug medication applied to the body, and others through discipline applied to the soul by means of punishments and reproaches; for by the encouragement they give strength and excite to an onward, movement, and exhort to beneficial deeds. The art of the gymnasium trainer, and its nearest approach, that of the medical man, do, on application to the body, reduce their powers to the utmost symmetry, purifying the blood, and equalizing the breath, so that, if there were there any diseased virulence, the powers of blood and breath may be vigorous; but music, and its leader, philosophy, which the laws and the gods ordained as regulators for the soul, accustom, persuade and partly compel the irrational to obey reason, and the two irrational, passion and desire, to become, the one mild, and the other quiet, so as not to be moved without reason, nor to be unmoved when the mind incites either to desire or enjoy something; for this is the definition of temperance, namely, docility and firmness. Intelligence and philosophy the highest in honor, after cleansing the soul from false opinions, have introduced knowledge, recalling the mind from excessive ignorance, and setting it free for the contemplation of divine things; in which to occupy oneself with self-sufficiency, as regards the affairs of a man, and with an abundance, for the commensurate period of life, is a happy state.

HUMAN DESTINY

Now he to whom the deity has happened to assign some what of a good fate, is, through opinion, led to the happiest life. But if he be morose and indocile, let the punishment that comes from law and reason follow him; bringing with it the fears ever on the stretch, both those that originate in heaven or Hades; how that punishments inexorable, are below laid up for the unhappy, as well as those ancient Homeric threats of retaliation for the wickedness of those defiled by crime (Odyssey, xii 571-599). For as we sometimes restore bodies to health by means of diseased substances, if they will not yield to the more healthy, so if the soul will not be led by true reasoning, we restrain it by false. Strange indeed would those punishments be called since, by a change, the souls of cowards enter into bodies of women, who are inclined to insulting conduct; and those of the blood-stained would be punished by being introduced into the bodies of wild beasts; of the lascivious, into the bodies of sows and boars; of the light-minded and frivolous into shaper and aeronautic birds; and of those who neither do learn or think of nothing, into the bodies of idle fish.

On all these matters, however, there has, at a second period, been delivered a judgment by Nemesis, or Fate, together with the avenging deities that preside over murderers, and those under the earth in Hades, and the inspectors of human affairs, to whom God, the leader of all, has

entrusted the administration of the world which is filled with gods and men, and the rest of the living beings which by the demiurgic creator according to the best model of an unbegotten, eternal and mentally-perceived form.

Pythagoras and the Pythagoreans, Fragments and Commentary

Arthur Fairbanks, ed. and trans.
The First Philosophers of Greece
(London: K. Paul, Trench, Trubner, 1898), 132-156.

Hanover Historical Texts Project
Scanned and proofread by Aaron Gulyas, May 1998.

Fairbanks's Introduction

(132)
Pythagoras, son of Mnesarchos, a native of Samos, left his fatherland to escape the tyranny of Polykrates (533/2 or 529/8 B.C.). He made his home for many years in Kroton in southern Italy, where his political views gained control in the city. At length he and his followers were banished by an opposing party, and he died at Metapontum. Many stories are told of his travels into Egypt and more widely, but there is no evidence on which the stories can be accepted. He was a mystic thinker and religious reformer quite as much as a philosopher, but there is no reason for denying that the doctrines of the school originated with him. Of his disciples, Archytas, in southern Italy, and Philolaos and Lysis, at Thebes, are the best known. It is the doctrine of the school, not the teaching of Pythagoras himself, which is known to us through the writings of Aristotle.

Passages in Plato referring to the Pythagoreans

(133) *Phaedo* 62 B. The saying that is uttered in secret rites, to the effect that we men are in a sort of prison, and that one ought not to loose himself from it nor yet to run away, seems to me something great and not easy to see through; but this at least I think is well said, that it is the gods who care for us, and we men are one of the possessions of the gods.

Kratyl. 400 B. For some say that it (the body) is the tomb of the soul-I think it was the followers of Orpheus in particular who introduced this word-which has this enclosure like a prison in order that it may be kept safe.

Gorg. 493 A. I once heard one of the wise men say that now we are dead and the body is our tomb, and that that part of the soul where desires are, it so happens, is open to persuasion, and moves upward or downward. And, indeed, a clever man-perhaps some inhabitant of Sicily or Italy-speaking allegorically, and taking the word from credible' and 'persuadable' called this a jar; and he called those without intelligence uninitiated, and that part of the soul of uninitiated persons where the desires are, he called its intemperateness, and said it was not water tight, as a jar might be pierced with holes-using the simile because of its insatiate desires.

Gorg. 507 A. And the wise men say that one community embraces heaven and earth and gods and men and friendship and order and temperance and righteousness, and for that reason they call this whole a universe, (134) my friend, for it is not without order nor yet is there excess. It

seems to me that you do not pay attention to these things, though you are wise in regard to them. But it has escaped your notice that geometrical equality prevails widely among both gods and men.

Passages in Aristotle referring to the Pythagoreans

Phys. iii. 4; 203 a 1. For all who think they have worthily applied themselves to such philosophy, have discoursed concerning the infinite, and they all have asserted some first principle of things-some, like the Pythagoreans and Plato, a first principle existing by itself, not connected with anything else, but being itself the infinite in its essence. Only the Pythagoreans found it among things perceived by sense (for they say that number is not an abstraction), and they held that it was the infinite outside the heavens.

iii. 4; 204 a 33. (The Pythagoreans) both hold that the infinite is being, and divide it. iv. 6; 213 b 22. And the Pythagoreans say that there is a void, and that it enters into the heaven itself from the infinite air, as though it (the heaven) were breathing; and this void defines the natures of things, inasmuch as it is a certain separation and definition of things that lie and this is true first in the case of numbers, for the void defines the nature of these.

De coel. i. ; 268 a 10. For as the Pythagoreans say, the all and all things are defined by threes; for end and middle and beginning constitute the number of the all, and also the number of the triad.

ii. 2; 284 b 6. And since there are some who say that there is a right and left of the heavens, as, for instance, (135) those that are called Pythagoreans (for such is their doctrine), we must investigate whether it is as they say.

ii. 2; 285 a 10. Wherefore one of the Pythagoreans might be surprised in that they say that there are only these two first principles, the right and the left, and they pass over four of them as not having the least validity; for there is no less difference up and down, and front and back than there is right and left in all creatures.

ii. 2; 285 b 23. And some are dwelling in the upper hemisphere and to the right, while we dwell below and to the left, which is the opposite to what the Pythagoreans say; for they put us above and to the right, while the others are below and at the left.

ii.9 ; 290 b 15. Some think it necessary that noise should arise when so great bodies are in motion, since sound does arise from bodies among us which are not so large and do not move so swiftly; and from the sun and moon and from the stars in so great number, and of so great size, moving so swiftly, there must necessarily arise a sound inconceivably great. Assuming these things and that the swiftness has the principle of harmony by reason of the intervals, they say that the sound of the stars moving on in a circle becomes musical. And since it seems unreasonable that we also do not hear this sound, they say that the reason for this is that the noise exists in the very nature of things, so as not to be distinguishable from the opposite silence; for the distinction of sound and silence lies in their contrast with each other, so that as blacksmiths think there is no

difference between them because they are accustomed to the sound, so the same thing happens to men.

ii. 9; 291 a 7. What occasions the difficulty and makes the Pythagoreans say that there is a harmony of the bodies as they move, is a proof. For whatever things

(136) move themselves make a sound and noise; but whatever things are fastened in what moves or exist in it as the parts in a ship, cannot make a noise, nor yet does the ship if it moves in a river.

ii. 13; 293 a 19. They say that the whole heaven is limited, the opposite to what those of Italy, called the Pythagoreans, say; for these say that fire is at the centre and that the earth is one of the stars, and that moving in a circle about the centre it produces night and day. And they assume yet another earth opposite this which they call the counter-earth, not seeking reasons and causes for phenomena, but stretching phenomena to meet certain assumptions and opinions of theirs and attempting to arrange them in a system. . . . And farther the Pythagoreans say that the most authoritative part of the All stands guard, because it is specially fitting that it should, and this part is the centre; and this place that the fire occupies, they call the guard of Zeus, as it is called simply the centre, that is, the centre of space and the centre of matter and of nature.

iii.1; 300 a 15. The same holds true for those who construct the heaven out of numbers; for some construct nature out of numbers, as do certain of the Pythagoreans.

Metaphys. i. 5 ; 985 b 23-986 b 8. With these and before them (Anaxagoras, Empedokles, Atomists) those called Pythagoreans applying themselves to the sciences, first developed them ; and being brought up in them they thought that the first principles of these (i.e. numbers) were the first principles of all things. And since of these (sciences) numbers are by nature the first, in numbers rather than in fire and earth and water they thought they saw many likenesses to things that are and that are coming to be, as, for instance, justice is such a property of numbers, and soul and mind are (137) such a property, and another is opportunity, and of other things one may say the same of each one.

And further, discerning in numbers the conditions and reasons of harmonies also; since, moreover, other things seemed to be like numbers in their entire nature, and numbers were the first of every nature, they assumed that the elements of numbers were the elements of all things, and that the whole heavens were harmony and number. And whatever characteristics in numbers and harmonics they could show were in agreement with the properties of the heavens and its parts and with its whole arrangement, these they collected and adapted; and if there chanced to be any gap anywhere, they eagerly sought that the whole system might be connected with these (stray phenomena). To give an example of my meaning: inasmuch as ten seemed to be the perfect number and to embrace the whole nature of numbers, they asserted that the number of bodies moving through the heavens were ten, and when only nine were visible, for the reason just stated they postulated the counter-earth as the tenth. We have given a more definite account of these thinkers in other parts of our writings. But we have referred to them here with this purpose in view, that we might ascertain from them what they asserted as the first principles and in what manner they came upon the causes that have been enumerated. They certainly seem to

consider number as the first principle and as it were the matter in things and in their conditions and states; and the odd and the even are elements of number, and of these the one is infinite and the other finite, and unity is the product of both of them, for it is both odd and even, and number arises from unity, and the whole heaven, as has been said, is numbers.

A different party in this same school say that the (138) first principles are ten, named according to the following table: -finite and infinite, even and odd, one and many, right and left, male and female, rest and motion, straight and crooked, light and darkness, good and bad, square and oblong. After this manner Alkmaeon of Kroton seems to have conceived them, and either he received this doctrine from them or they from him ; for Alkmaeon arrived at maturity when Pythagoras was an old man, and his teachings resembled theirs. For he says that most human affairs are twofold, not meaning opposites reached by definition, as did the former party, but opposites by chance - as, for example, white-black, sweet-bitter, good-bad, small-great. This philosopher let fall his opinions indefinitely about the rest, but the Pythagoreans declared the number of the opposites and what they were. From both one may learn this much, that opposites are the first principles of things; but from the latter he may learn the number of these, and what they are. But how it is possible to bring them into relation with the causes of which we have spoken if they have not clearly worked out; but they seem to range their elements under the category of matter, for they say that being is compounded and formed from them, and that they inhere in it.

987 a 9-27. Down to the Italian philosophers and with the exception of them the rest have spoken more reasonably about these principles, except that, as we said, they do indeed use two principles, and the one of these, whence is motion, some regard as one and others as twofold. The Pythagoreans, however, while they in similar manner assume two first principles, add this which is peculiar to themselves: that they do not think that the finite and the infinite and the one are certain other things by nature, such as fire or earth or any other such thing, but the infinite itself and unity itself are (139) the essence of the things of which they are predicated, and so they make number the essence of all things. So they taught after this manner about them, and began to discourse and to define what being is, but they made it altogether too simple a matter. For they made their definitions superficially, and to whatever first the definition might apply, this they thought to be the essence of the matter ; as if one should say that twofold and two were the same, because the twofold subsists in the two. But undoubtedly the two and the twofold are not the same; otherwise the one will be many - a consequence which even they would not draw. So much then may be learned from the earlier philosophers and from their successors.

i. 6; 987 b 10. And Plato only changed the name, for the Pythagoreans say that things exist by imitation of numbers, but Plato, by sharing the nature of numbers.

i. 6; 987 b 22. But that the one is the real essence of things, and not something else with unity as an attribute, he affirms, agreeing with the Pythagoreans; and in harmony with them he affirms that numbers are the principles of being for other things. But it is peculiar to him that instead of a single infinite he posits a double infinite, an infinite of greatness and of littleness; and it is also peculiar to him that he separates numbers from things that are seen, while they say that numbers are the things themselves, and do not interpose mathematical objects between them. This

separation of the one and numbers from things, in contrast with the position of the Pythagoreans, and the introduction of ideas, are the consequence of his investigation by concepts.

i. 8; 989 b 32-990 a 32. Those, however, who carry on their investigation with reference to all things, and divide things into what are perceived and what are not perceived by sense, evidently examine both classes, so (140) one must delay a little longer over what they say. They speak correctly and incorrectly in reference to the questions now before us. Now those who are called Pythagoreans use principles and elements yet stranger than those of the physicists, in that they do not take them from the sphere of sense, for mathematical objects are without motion, except in the case of astronomy. Still, they discourse about everything in nature and study it they construct the heaven, they observe what happens in its parts and their states and motions; they apply to these their first principles and causes, as though they agreed entirely with the other physicists that being is only what is perceptible and what that which is called heaven includes. But their causes and first principles, they say, are such as to lead up to the higher parts of reality, and are in harmony with this rather than with the doctrines of nature. In what manner motion will take place when finite and infinite, odd and even, are the only underlying realities, they do not say; nor how it is possible for genesis and destruction to take place without motion and change, or for the heavenly bodies to revolve. Farther, if one grant to them that greatness arises from these principles, or if this could be proved, nevertheless, how will it be that some bodies are light and some heavy ? For their postulates and statements apply no more to mathematical objects than to things of sense; accordingly they have said nothing at all about fire or earth or any such objects, because I think they have no distinctive doctrine about things of sense. Farther, how is it necessary to assume that number and states of number are the causes of what is in the heavens and what is taking place there from the beginning and now, and that there is no other number than that out of which the world is composed? For when opinion and opportune time are at a certain point in the heavens, (141) and a little farther up or down are injustice and judgment or a mixture of them, and they bring forward as proof that each one of these is number, and the result then is that at this place there is already a multitude of compounded quantities because those states of number have each their place-is this number in heaven the same which it is necessary to assume that each of these things is, or is it something different? Plato says it is different ; still, he thinks that both these things and the causes of them are numbers; but the one class are ideal causes, and the others are sense causes.

ii. 1 ; 996 a 4. And the most difficult and perplexing question of all is whether unity and being are not, as Plato and the Pythagoreans say, something different from things but their very essence, or whether the underlying substance is something different, friendship, as Empedokles says, or as another says, fire, or water, or air.

ii. 4 ; 1001 a 9. Plato and the Pythagoreans assert that neither being nor yet unity is something different from things, but that it is the very nature of them, as though essence itself consisted in unity and existence.

1036 b 17. So it turns out that many things of which the forms appear different have one form, as the Pythagoreans discovered; and one can say that there is one form for everything, and the others are not forms; and thus all things will be one.

ix. 2 ; 1053 b 11. Whether the one itself is a sort of essence, as first the Pythagoreans and later Plato, affirmed.

xi. 7; 1072 b 31. And they are wrong who assume, as do the Pythagoreans and Speusippos, that the most beautiful and the best is not in the first principle, because the first principles of plants and animals are indeed causes; for that which is beautiful and perfect is in what comes from these first principles.

(142) xii. 4; 1078 b 21. The Pythagoreans (before Demokritos) only defined a few things, the concepts of which they reduced to numbers, as for instance opportunity or justice or marriage. . .

xii. 6; 1080 b 16. The Pythagoreans say that there is but one number, the mathematical, but things of sense are not separated from this, for they are composed of it; indeed, they construct the whole heaven out of numbers, but not out of unit numbers, for they assume that the unities have quantity; but how the first unity was so constituted as to have quantity, they seem at a loss to say. b 31. All, as many as regard the one as the element and first principle of things, except the Pythagoreans, assert that numbers are based on the unit; but the Pythagoreans assert, as has been remarked, that numbers have quantity.

xii. 8; 1083 b 9. The Pythagorean standpoint has on the one hand fewer difficulties than those that have been discussed, but it has new difficulties of its own. The fact that they do not regard number as separate, removes many of the contradictions ; but it is impossible that bodies should consist of numbers, and that this number should be mathematical. Nor is it true that indivisible elements have quantity; but, granted that they have this quality of indivisibility, the units have no quantity; for how can quantity be composed of indivisible elements? but arithmetical number consists of units. But these say that things are number; at least, they adapt their speculations to such bodies as consist of elements which are numbers.

xiii. 3; 1090 a 20. On the other hand the Pythagoreans, because they see many qualities of numbers in bodies perceived by sense, regard objects as numbers, not as separate numbers, but as derived from numbers. And why? Because the qualities of numbers exist in (143) harmony both in the heaven and in many other things. But for those who hold that number is mathematical only, it is impossible on the basis of their hypothesis to say any such thing; and it has already been remarked that there can be no science of these numbers. But we say, as above, that there is a science of numbers. Evidently the mathematical does not exist apart by itself, for in that case its qualities could not exist in bodies. In such a matter the Pythagoreans are restrained by nothing ; when, however, they construct out of numbers physical bodies out of numbers that have neither weight nor lightness, bodies that have weight and lightness - they seem to be speaking about another heaven and other bodies than those perceived by sense.

Eth. i. 4 ; 1096 b 5. And the Pythagoreans seem to speak more persuasively about it, putting the unity in the co-ordination of good things.

ii. 5; 1106 b 29. The evil partakes of the nature of the infinite, the good of the finite, as the Pythagoreans conjectured.

v. 8; 1132 b 21. Reciprocity seems to some to be absolutely just, as the Pythagoreans say; for these defined the just as that which is reciprocal to another.

Mor. i. 1; 1182 a 11. First Pythagoras attempted to speak concerning virtue, but he did not speak correctly for bringing virtues into correspondence with numbers, he did not make any distinct.

Pythagoras and the Pythagoreans: Passages in the Doxographists

Aet. *Plac*. i. 3; `. 280. And again from another starting-point, Pythagoras, son of Muesarchos, a Samian, who was the first to call this matter by the name of philosophy, assumed as first principles the numbers an (144) the symmetries existing in them, which he calls harmonies, and the elements compounded of both, that are called geometrical. And again he includes the monad and the undefined dyad among the first principles; and for him one of the first principles tends toward the creative and form-giving cause, which is intelligence, that is god, and the other tends toward the passive and material cause, which is the visible universe. And he says that the starting-point of number is the decad; for all Greeks and all barbarians count as far as ten, and when they get as far as this they return to the monad. And again, he says, the power of the ten is in the four and the tetrad. And the reason is this: if any one returning from the monad adds the numbers in a series as far as the four, he will fill out the number ten (i.e. 1 + 2 + 3 + 4 10); but if he goes beyond the number of the tetrad, he will exceed the ten. Just as if one should add one and two and should add to these three and four, he will fill out the number ten; so that according to the monad number is in the ten, but potentially in the four. Wherefore the Pythagoreans were wont to speak as though the greatest oath were the tetrad: 'By him that transmitted to our soul the tetraktys, which has the spring and root of ever-flowing nature.' And our soul, he says, is composed of the tetrad ; for it is intelligence, understanding, opinion, sense, from which things come every art and science, and we ourselves become reasoning beings. The monad, however, is intelligence, for intelligence sees according to the monad. As for example, men are made up of many parts, and part by part they are devoid of sense and comprehension and experience, yet we perceive that man as one alone, whom no being resembles, possesses these qualities; and we perceive that a horse is one, but part by part it is without experience. (145) For these are all forms and classes according to monads. Wherefore, assigning this limit with reference to each one of these, they speak of a reasoning being and a neighing being. On this account the monad is intelligence by which we perceive these things. And the undefined dyad is science; fittingly, for all proof and all persuasion is part of science, and farther every syllogism brings together what is questioned out of some things that are agreed upon, and easily proves something else; and science is the comprehension of these things, wherefore it would be the dyad. And opinion as the result of comprehending them is the triad; fittingly, for opinion has to do with many things; and the triad is quantity, as 'The thrice-blessed Danaoi.' On this account then he includes the triad. . . . And their sect is called Italic because Pythagoras taught in Italy, for he removed from Samos, his fatherland, because of dissatisfaction with the tyranny of Polykrates.

Act. i. 7 ; *Dox*. 302. Pythagoras held that one of the first principles, the monad, is god and the good, which is the origin of the One, and is itself intelligence; but the undefined dyad is a divinity and the bad, surrounding which is the mass of matter. i. 8 ; 307. Divine spirits are psychical beings; and heroes are souls separated from bodies, good heroes are good souls, bad heroes are bad souls. i. 9 ; 307. The followers of Thales and Pythagoras and the Stoics held that matter is variable and changeable and transformable and in a state of flux, the whole through the whole. i. 10; 309. Pythagoras asserted that the so-called forms and ideas exist in numbers and their harmonies, and in what are called geometrical objects, apart from bodies. i. 11 ; 310. Pythagoras and Aristotle asserted that the first causes are immaterial, but that other causes involve a union or contact with material substance [so that the world is (146) material]. i. 14; 312. The followers of Pythagoras held that the universe is a sphere according to the form of the four elements; but the highest fire alone is conical. i. 15; 314. The Pythagoreans call colour the manifestafion of matter. i. 16; 314. Bodies are subject to change of condition, and are divisible to infinity. i. 18; 316. (After quotation from Arist. *Phys*. iv. 4; 212 a 20) And in his first book on the philosophy of Pythagoras he writes that the heaven is one, and that time and wind and the void which always defines the places of each thing, are introduced from the infinite. And among other things he says that place is the immovable limit of what surrounds the world, or that in which bodies abide and are moved; and that it is full when it surrounds body on every side, and empty when it has absolutely nothing in itself. Accordingly it is necessary for place to exist, and body; and it is never empty except only from the standpoint of thought, for the nature of it in perpetuity is destructive of the interrelation of things and of the combination of bodies; and motions arise according to place of bodies that surround and oppose each other; and no infiniteness is lacking, either of quantity or of extent. i. 20; 318. Pythagoras said that time is the sphere of what surrounds the world. i. 21 ; 318. Pythagoras, Plato: Motion is a certain otherness or difference in matter. [This is the common limit of all motion.] i. 24; 320. Pythagoras and all that assume that matter is subject to change assert that genesis and destruction in an absolute sense take place for from change of the elements and modification and separation of them there take place juxtaposition and mixture, and intermingling and melting together. Aet. *Plac*. ii. 1; 327. Pythagoras first named the circumference of all things the universe by reason of the order in it. ii. 4; 330. Pythagoras, Plato, and the Stoics (147) held that the universe is brought into being by god. And it is perishable so far as its nature is concerned, for it is perceived by sense, and therefore material; it will not however be destroyed in accordance with the foreknowledge and plan of god. ii. 6; 334. Pythagoras: The universe is made from five solid figures, which are called also mathematical; of these he says that earth has arisen from the cube, fire from the pyramid, air from the octahedron, and water from the icosahedron, and the sphere of the all from the dodecahedron. ii. 9; 338. The followers of Pythagoras hold that there is a void outside the universe into which the universe breathes forth, and from which it breathes in. ii. 10; 339. Pythagoras, Plato, Aristotle: The right hand side of the universe is the eastern part from which comes the beginning of motion, and the left hand side is the west. They say the universe has neither height nor depth, in which statement height means distance from below upwards, and depth from above downwards. For none of the distances thus described exist for the universe, inasmuch as it is disposed around the middle of itself, from which it extends toward the all, and with reference to which it is the same on every side. ii. 12 ; 340. Thales, Pythagoras, and their followers: The sphere of the whole heaven is divided into five circles, which they call zones; the first of these is called the arctic zone and is ever visible; the second the summer solstice, the third the equinoctial, the fourth the winter solstice, and fifth the antarctic zone, which is invisible. And

the ecliptic called the zodiac in the three middle ones is projected to touch the three middle ones. And the meridian crosses all these from the north to the opposite quarter at right angles. It is said that Pythagoras was the first to recognise the slant of the zodiacal circle which Oenopides of Chios appropriated as his own (148) discovery. ii. 13; 343. Herakleides and the Pythagoreans asserted that each world [GREEK] of the stars is air and aether surrounding earth in the infinite aether. And these doctrines are brought out in the Orphic writings, for they construct each world of the stars. ii. 22; 352. The Pythagoreans: The sun is spherical. ii. 23; 353. Plato, Pythagoras, Aristotle: The solstices lie along the slant of the zodiacal circle, through which the sun goes along the zodiac, and with the accompaniment of the tropic circles; and all these things also the globe shows. 11.24; 354. An eclipse takes place when the moon comes past. ii. 25; 357. Pythagoras: The moon is a mirrorlike body. ii. 29; 360. Some of the Pythagoreans (according to the Aristotelian account and the statement of Philip the Opuntian) said that an eclipse of the moon takes place, sometimes by the interposition of the earth, sometimes by the interposition of the counter- earth. But it seems to some more recent thinkers that it takes place by a spreading of the flame little by little as it is gradually kindled, until it gives the complete full moon, and again, in like manner it grows less until the conjunction, when it is completely extinguished. ii. 30; 361. Some of the Pythagoreans, among them Philolaos, said that the earthy appearance of the moon is due to its being inhabited by animals and by plants, like those on our earth, only greater and more beautiful; for the animals on it are fifteen times as powerful, not having any sort of excrement, and their day is fifteen times as long as ours. But others said that the outward appearance in the moon is a reflection on the other side of the inflamed circle of the sea that is on our earth. ii. 32 ; 3(34. Some regard the greater year . . . . as the sixty year period, among whom are Ocuopides and Pythagoras.

Act. *Plac*. iii. 1; *Dox*. 364. Some of the Pythagoreans. (149) said that the milky way is the burning of a star that fell from its own foundation, setting on fire the region through which it passed in a circle, as Phaethon was burned. And others say that the course of the sun arose in this manner at the first. And certain ones say that the appearance of the sun is like a mirror reflecting its rays toward the heaven, and therefore it happens at times to reflect its rays on the rainbow in the clouds.

Aet. iii. 2; 366. Some of the followers of Pythagoras say that the comet is one of the stars that are not always shining, but emit their light periodically through a certain definite time; but others say that it is the reflection of our vision into the sun, like reflected images. iii. 14; 378. Pythagoras: The earth, after the analogy of the sphere of the all, is divided into five zones, arctic, antarctic, summer, winter, and equinoctial of these the middle one he defines to be the middle of the earth, called for this very reason the torrid zone; but the inhabited one [the one between the arctic and the torrid zone] being well-tempered.

Aet. iv. 2; *Dox*. 386. Pythagoras holds that number moves itself, and he takes number as an equivalent for intelligence. iv. 4; 389. Pythagoras, Plato: According to a superficial account the soul is of two parts, the one possessing, the other lacking, reason; but according to close and exact examination, of three parts; for the unreasoning part they divide into the emotions and the desires. (Theodor. v. 20); *Dox*. 390. The successors of Pythagoras saying that body is a mixture of five elements (for they ranked the aether as a fifth along with the four) held that the powers of the soul are of the same number as these. And these they name intelligence and wisdom and

understanding and opinion and sense-perception. iv. 5 ; 391. Pythagoras: The principle of life is about the heart, but the principle of reason and (150) intelligence is about the head. iv. 5; 392. Pythagoras et al. : The intelligence enters from without. iv. 7; 392. Pythagoras, Plato : The soul is imperishable. iv. 9; 396. Pythagoras et al. : The sense-perceptions are deceptive. iv. 9 ; 397. Pythagoras, Plato: Each of the sensations is pure, proceeding from each single element. With reference to vision, it was of the nature of aether hearing, of the nature of wind; smell, of the nature of fire ; taste, of the nature of moisture; touch, of the nature of earth. iv. 14; 405. The followers of Pythagoras and of the mathematicians on reflections of vision For vision moves directly as it were against the bronze [of a mirror], and meeting with a firm smooth surface it is turned and bent back on itself, meeting some such experience as when the arm is extended and then bent back to the shoulder. iv. 20; 409. Pythagoras, Plato, Aristotle: Sound is immaterial. For it is not air, but it is the form about the air and the appearance after some sort of percussion which becomes sound; and every appearance is immaterial; for it moves with bodies, but is itself absolutely immaterial; as in the case of a bent rod the surface-appearance suffers no change, but the matter is what is bent.

Aet. *Plac*. v.1; 415. Pythagoras did not admit the sacrificial part alone (of augury). v. 3 ; 417. Pythagoras : The seed is foam of the best part of the blood, a secretion from the nourishment, like blood and marrow. v.4; 417. Pythagoras, Plato, Aristotle: The power of seed is immaterial, like intelligence, the moving power but the matter that is poured forth is material. v.20; 432. Pythagoras, Plato: The souls of animals called unreasoning are reasonable, not however with active reasoning powers, because of an imperfect mixture of the bodies and because they do not have the power of (151) speech, as in the case of apes and dogs; for these have intelligence but not the power of speech.

Ar. Did. *Ep*. Fr. 32; Dox. 467. Apollodoros in the second book Concerning the gods: It is the Pythagorean opinion that the morning and the evening star are the same.

Theophr. *Phys. Op*. Fr. 17; Dox. 492. Favorinus says that he (Pythagoras) was the first to call the heavens a universe and the earth round.

Cic. de *Deor*. Nat. i. 11 ; Philod. piet. Fr. c 4 b; *Dox.* 533. For Pythagoras, who held that soul is extended through all the nature of things and mingled with them, and that from this our souls are taken, did not see that god would be separated and torn apart by the separation of human souls; and when souls are wretched, as might happen to many, then part of god would be wretched; a thing which could not happen.

Hippol. *Phil*. 2 ; *Dox.* 555. There is a second philosophy not far distant from the same time, of which Pythagoras, whom some call a Samian, was the first representative. And this they call the Italian philosophy because Pythagoras fled the rule of Polykrates over the Samians and settled in a city of Italy where he spent his life. The successive leaders of this sect shared the same spirit. And he in his studies of nature mingled astronomy and geometry and music . And thus he asserted that god is a monad, and examining the nature of number with especial care, he said that the universe produces melody and is put together with harmony, and he first proved the motion of the seven stars to be rhythm and melody. And in wonder at the structure of the universe, he decreed that at first his disciples should be silent, as it were mystae who were coming into the

order of the all; then when he thought they had sufficient education (152) in the principles of truth, and had sought wisdom sufficiently in regard to stars and in regard to nature, he pronounced them pure and then bade them speak. He separated his disciples into two groups, and called one esoteric, and the other exoteric. To the former he entrusted the more perfect sciences, to the latter the more moderate. And he dealt with magic, as they say, and himself discovered the art of physiognomy. Postulating both numbers and measures he was wont to say that the first principle of arithmetic embraced philosophy by combination, after the following manner:

Number is the first principle, a thing which is undefined, incomprehensible, having in itself all numbers which could reach infinity in amount. And the first principle of numbers is in substance the first monad, which is a male monad, begetting as a father all other numbers. Secondly the dyad is female number, and the same is called by the arithmeticians even. Thirdly the triad is male number; this the arithmeticians have been wont to call odd. Finally the tetrad is a female number, and the same is called even because it is female.

All numbers, then, taken by classes are fours (for number is undefined in reference to class), of which is composed the perfect number, the decad. For the series, one two three and four, becomes ten, if its own name is kept in its essence by each of the numbers. Pythagoras said that this sacred tetraktys is 'the spring having the roots of ever-flowing nature in itself, and from this numbers have their first principle. For the eleven and the twelve and the rest derive from the ten the first principle of their being. The four parts of the decad, this perfect number, are called number, monad, power, and cube. And the interweavings and (153) minglings of these in the origin of growth are what naturally completes nascent number; for when a power is multiplied upon itself, it is the power of a power and when a power is multiplied on a cube, it is the power of a cube ; and when a cube is multiplied on a cube, the cube of a cube; thus all numbers, from which arises the genesis of what arises, are seven: -number, monad, power, cube, power of a power, power of a cube, cube of a cube.

He said that the soul is immortal, and that it changes from one body to another[Cf. Epiph. Haer. i. 7; Dox. 589.]; so he was wont to say that he himself had been born before the Trojan war as Aethalides, and at the time of the Trojan war as Euphorbos, and after that as Hermotimos of Samos, then as Pyrrhos of Delos, fifth as Pythagoras. And Diodoros of Eretria and Aristoxenos the musician say that Pythagoras had come into Zaratas of Chaldaea and he set forth that in his view there were from the beginning two causes of things, father and mother and the father is light and the mother darkness; and the parts of light are warm, dry, light, swift; and of darkness are cold, moist, heavy, slow ; and of these the universe is composed, of male and female. And he says that the universe exists in accordance with musical harmony, so the sun also makes an harmonious period. And concerning the things that arise from the earth and the universe they say that Zaratas spoke as follows: There are two divinities, one of the heavens and the other of the earth; the one of the earth produces things from the earth, and it is water; and the divinity of the heavens is fire with a portion of air, warm, and cold; wherefore he says that none of these things will destroy or even pollute the soul, for these are the essence of all things. And it is said that Zaratas forbade men (154) to eat beans because he said that at the beginning and composition of all things when the earth was still a whole, the bean arose. And he says that the proof of this is that if one chews a bean to a pulp and exposes it to the sun for a certain time (for the sun will affect it quickly), it gives out the odour of human seed. And he says that there is another and

clearer proof: if when a bean is in flower we were to take the bean and its flower, and putting it into a pitcher moisten it and then bury it in the earth, and after a few days dig it up again, we should see in the first place that it had the form of a womb, and examining it closely we should find the head of a child growing with it. He perished in a conflagration with his disciples in Kroton in Italy. And it was the custom when one became a disciple for him to burn his property and to leave his money under a seal with Pythagoras, and he remained in silence sometimes three years, sometimes five years, and studied. And immediately on being released from this he mingled with the others and continued a disciple and made his home with them; otherwise he took his money and was sent off. The esoteric class were called Pythagoreans, and the others Pythagoristae. And those of the disciples who escaped the conflagration were Lysis and Archippos and Zalmoxis the slave of Pythagoras, who is said to have taught the Pythagorean philosophy to the Druids among the Celts.1 It is said that Pythagoras learned numbers and measures from the Egyptians; astonished at the wisdom of the priests, which was deserving of belief and full of fancies and difficult to buy, lie imitated it and himself also taught his disciples to be silent, and obliged the student to remain quietly in rooms underneath the earth.

Epiph. *Pro.* i.; *Dox.* 587. Pythagoras laid down (155) the doctrine of the monad and of foreknowledge and the interdict on sacrificing to the gods then believed on, and he bade men not to partake of beings that had life, and to refrain from wine. And he drew a line between the things from the moon upwards, calling these immortal, and those below, which he called mortal; and he taught the transmigration of souls from bodies into bodies even as far as animals and beasts. And he used to teach his followers to observe silence for a period of five years. Finally he named himself a god.

Epiph. Haer. iii. 8 ; Dox. 390. Pythagoras the Samian, son of Mnesarchos, said that the monad is god, and that nothing has been brought into being apart from this. He was wont to say that wise men ought not to sacrifice animals to the gods, nor yet to eat what had life, or beans, nor to drink wine. And he was wont to say that all things from the moon downward were subject to change, while from the moon upward they were not. And he said that the soul goes at death into other animals. And he bade his disciples to keep silence for a period of five years, and finally he named himself a god.

Herm. *I.G.P*. 16; *Dox*. 655. Others then from the ancient tribe, Pythagoras and his fellow-tribesmen, revered and taciturn, transmitted other dogmas to me as mysteries, and this is the great and unspeakable ipscedixit: the monad is the first principle of all things. From its forms and from numbers the elements arose. And he declared that the number and form and measure of each of these is somehow as follows: -Fire is composed of twenty-four right-angled triangles, surrounded by four equilaterals. And each equilateral consists of six right-angled triangles, whence they compare it to the pyramid. Air is composed of forty-eight triangles, stir-rounded by eight equilaterals. And it is compared to (156) the octahedron, which is surrounded by eight equilateral triangles, each of which is separated into six right-angled triangles so as to become forty-eight in all. And water is composed of one hundred and twenty triangles, surrounded by twenty equilaterals, and it is compared to the icosahedron, which is composed of one hundred and twenty equilateral triangles. And aether is composed of twelve equilateral pentagons, and is like a dodecahedron. And earth is composed of forty-eight triangles, and is surrounded by six

equilateral pentagons, and it is like a cube. For the cube is surrounded by six tetragons, each of which is separated into eight triangles, so that they become in all forty-eight.

Pythagoreanism

The philosophical school and religious brotherhood known as Pythagoreanism is believed to have been founded by Pythagoras of Samos, who settled in Croton in southern Italy about 525 BC.

GENERAL FEATURES OF PYTHAGOREANISM

The character of the original Pythagoreanism is controversial, and the conglomeration of disparate features that it displayed is intrinsically confusing. Its fame rests, however, on some very influential ideas, not always correctly understood, that have been ascribed to it since antiquity. These ideas include those of (1) the metaphysic of number and the conception that reality, including music and astronomy, is, at its deepest level, mathematical in nature; (2) the use of philosophy as a means of spiritual purification; (3) the heavenly destiny of the soul and the possibility of its rising to union with the divine; (4) the appeal to certain symbols, sometimes mystical, such as the *tetraktys*, the golden section, and the harmony of the spheres (to be discussed below); (5) the Pythagorean theorem; and (6) the demand that members of the order shall observe a strict loyalty and secrecy. (see also number system)

By laying stress on certain inner experiences and intuitive truths revealed only to the initiated, Pythagoreanism seems to have represented a soul-directed subjectivism alien to the mainstream of Pre-Socratic Greek thought centring on the Ionian coast of Asia Minor (Thales, Anaximander, Anaxagoras, and others), which was preoccupied with determining what the basic cosmic substance is. (see also Ionian school)

In contrast with such Ionian naturalism, Pythagoreanism was akin to trends seen in mystery religions and emotional movements, such as Orphism, which often claimed to achieve through intoxication a spiritual insight into the divine origin and nature of the soul. Yet there are also aspects of it that appear to have owed much to the more sober, "Homeric" philosophy of the Ionians. The Pythagoreans, for example, displayed an interest in metaphysics (the nature of Being), as did their naturalistic predecessors, though they claimed to find its key in mathematical form rather than in any substance. They accepted the essentially Ionian doctrines that the world is composed of opposites (wet-dry, hot-cold, etc.) and generated from something unlimited; but they added the idea of the imposition of limit upon the unlimited and the sense of a musical harmony in the universe. Again, like the Ionians, they devoted themselves to astronomical and geometrical speculation. Combining, as it does, a rationalistic theory of number with a mystic numerology and a speculative cosmology with a theory of the deeper, more enigmatic reaches of the soul, Pythagoreanism interweaves Rationalism and irrationalism more inseparably than does any other movement in ancient Greek thought. (see also opposites, table of)

MAJOR CONCERNS AND TEACHINGS

The problem of describing Pythagoreanism is complicated by the fact that the surviving picture is far from complete, being based chiefly on a small number of fragments from the time before Plato and on various discussions in authors who wrote much later--most of whom were either Aristotelians or Neoplatonists (see below *History of Pythagoreanism*). In spite of the historical uncertainties, however, that have plagued searching scholars, the contribution of Pythagoreanism to Western culture has been significant and therefore justifies the effort, however inadequate, to

depict what its teachings may have been. Moreover, the heterogeneousness of Pythagorean doctrines has been well documented ever since Heracleitus, a classic early 5th-century Greek philosopher who, scoffing at Pythagoras' wide-ranging knowledge, said that it "does not teach one to have intelligence." There probably never existed a strictly uniform system of Pythagorean philosophy and religious beliefs, even if the school did have a certain internal organization. Pythagoras appears to have taught by pregnant, cryptic *akousmata* ("something heard") or *symbola*. His pupils handed these on, formed them partly into *Hieroi Logoi* ("Sacred Discourses"), of which different versions were current from the 4th century on, and interpreted them according to their convictions.
Western Philosophical Schools and Doctrines

Religion and ethics.

The belief in the transmigration of souls provided a basis for the Pythagorean way of life. Some Pythagoreans deduced from this belief the principle of "the kinship of all beings," the ethical implications of which were later stressed in 4th-century speculation. Pythagoras himself seems to have claimed a semidivine status in close association with the superior god Apollo; he believed that he was able to remember his earlier incarnations and, hence, to know more than others knew. Recent research has emphasized shamanistic traits deriving from the ecstatic cult practices of Thracian medicine men in the early Pythagorean outlook. The rules for the religious life that Pythagoras taught were largely ritualistic: refrain from speaking about the holy, wear white clothes, observe sexual purity, do not touch beans, and so forth. He seems also to have taught purification of the soul by means of music and mental activity (later called philosophy) in order to reach higher incarnations. "To be like your Master" and so "to come nearer to the gods" was the challenge that he imposed on his pupils. Salvation, and perhaps ultimate union with the divine cosmos through the study of the cosmic order, became one of the leading ideas in his school. (see also reincarnation)

The advanced ethics and political theories sometimes ascribed to Pythagoreanism may to some extent reflect ideas later developed in the circle of Archytas, the leading 4th-century Pythagorean. But a picture current among the Peripatetics (the school founded by Aristotle) of Pythagoras as the educator of the Greeks, who publicly preached a gospel of humanity, is clearly anachronistic. Several of the Peripatetic writers, Aristoxenus, Dicaearchus, and Timaeus, seem to have interpreted some principles--properly laid down only for esoteric use in the brotherhood--as though these applied to all mankind: the internal loyalty, modesty, self-discipline, piety, and abstinence required by the secret doctrinal system; the higher view of womanhood reflected in the admission of women to the school; a certain community of property; and perhaps the drawing of a parallelism between the macrocosm (the universe) and the microcosm (man), in which (for instance) the Pythagorean idea that the cosmos is an organism was applied to the state, which should thus mix monarchy, oligarchy, and democracy into a harmonic whole--these were all universalized.

PYTHAGORAS

PYTHAGOREAN SYMBOLS, or MAXIMS

(From Hierocles.)

1. Go not beyond the balance. (Transgress not Justice).

2. Sit not down on the bushel. (Do not loaf on your job).

3. Tear not to pieces the crown. (Do not be a joy-killer).

4. Eat not the heart. (Do not grieve over-much).

5. Do not poke the fire with a sword. (Do not further inflame the quarrelsome).

6. Having arrived at the frontiers, turn not back. (Do not wish to live over your life).

7. Go not by the public way. (Go not the broad popular way, which leads to destruction).

8. Suffer not swallows around your house. (Receive no swallows in your family).

9. Wear not the image of God on your ring. (Profane not the name of God).

10. Do not unload people, but load them up. (Encourage not idleness, but virtue).

11. Not easily shake hands with a man. (Make no ill-considered friendships).

12. Leave no the least mark of the pot on the ashes. (After reconciliation, forget the disagreement).

13. Sow mallows, but never eat them. (Use mildness to others, but not to yourself).

14. Wipe not out the place of the torch. (Let not all the lights of reason be extinguished).

15. Wear not a narrow ring. (Seek freedom, avoid slavery).

16. Feed not the animals that have crooked claws. (To your family admit no thief or traitor).

17. Abstain from beans. (Avoid farcineous food causing flatulence, avoid democratic voting).

18. Eat not fish whose tails are black. (Frequent not the company of men without reputation).

19. Never eat the gurnet. (Avoid revenge).

20. Eat not the womb of animals. (Avoid what leads to generation, to lowest affections).

21. Abstain from flesh of animals that die of themselves. (Avoid decayed food).

22. Abstain from eating animals. (Have no conversation with unreasonable men).

23. Always put salt on the table. (Always use the principle of Justice to settle problems).

24. Never break the bread. (When giving charity, do not pare too close).

25. Do not spill oil upon the seat. (Do not flatter princes, praise God only).

26. Put not meat in a foul vessel. (Do not give good precepts to a vicious soul).

27. Feed the cock, but sacrifice him not; for he is sacred to the sun and the moon. (Cherish people who warm you, sacrifice them not to resentment).

28. Break not the teeth. Do not revile bitterly. (Do not be sarcastic).

29. Keep far from you the vinegar-cruet. (Avoid malice and sarcasm).

30. Spit upon the parings of your nails, and on the clippings of your hair. (Abhor desires).

31. Do not urinate against the sun. (Be modest).

32. Speak not in the face of the sun. (Make not public the thoughts of your heart).

33 Do not sleep at noon. (Do not continue in darkness).

34. Stir up the bed as soon as you are risen, do not leave in it any print of the body. (When working, hanker not for luxurious ease).

35. Never sing without harp-accompaniment. (Make of life a whole).

36. Always keep your things packed up. (Always be prepared for all emergencies).

37. Quit not your post without your general's order. (Do. not suicide).

38. Cut not wood on the public road. (Never turn to private use what belongs to the public).

39. Roast not what is boiled. (Never take in ill part what is done in simplicity and ignorance).

40. Avoid the two-edged sword. (Have no conversation with slanderers).

41. Pick not up what is fallen from the table.(Always leave something for charity).

42. Abstain even from a cypress chest. (Avoid going to funerals).

43. To the celestial gods sacrifice an odd number, but to the infernal, an even. (To God consecrate the indivisible soul, the body to hell).

44. Offer not to the gods the wine of an unpruned vine. (Agriculture is a great piece of piety).

45. Never sacrifice without meal. (Encourage agriculture, offer bloodless offerings).

46. Adore the gods, and sacrifice bare-foot. (Pray and sacrifice in humility of heart).

47. Turn round when you worship. (Adore the immensity of God, who fills the universe).

48. Sit down when you worship. (Never worship in a hurry).

49. Pare not your nails during the sacrifices. (In the temple behave respectfully).

50. When it thunders, touch the ground. (Appease God by humility).

51. Do not primp by torch-light. (Look at things in the light of God).

52. One, Two. (God and Nature; all things are known to God).

53. Honor marks of dignity, the Throne, and the Ternary. (Worship magistrates, Kings, Heroes, Geniuses and God).

54. When the winds blow, adore echo. (During revolts, flee to deserts).

55. Eat not in the chariot. (Eat not in the midst of hurried, important business).

56. Put on your right shoe first, and wash your left foot first.(Prefer an active life, to one of ease and pleasure).

57. Eat not the brain. (Wear not out the brain, refresh yourself).

58. Plant not the palm-tee. (Do nothing but what is good and useful).

59. Make thy libations to the gods by the ear. (Beautify thy worship by music).

60. Never catch the cuttle-fish. (Undertake no dark affairs, intricate affairs, that will wound you).

61.Stop not at the threshold. (Be not wavering but choose your side).

62. Give way to a flock that goes by. (Oppose not the multitude).

63. Avoid the weasel. (Avoid tale-tellers).

64. Refuse the weapons a woman offers you. (Reject all suggestions revenge inspires).

65. Kill not the serpent that chances to fall within your walls. (Harm no enemy who becomes your guest or suppliant).

66. It is a crime to throw stones into fountains. (It is a crime to persecute good men)

67. Feed not yourself with your left hand. (Support yourself with honest toil, not robbery).

68. It is a horrible crime to wipe off the sweat with iron. (It is a criminal to deprive a man by force of what he earned by labor).

69. Stick not iron in the footsteps of a man. (Mangle not the memory of a man).

70. Sleep not on a grave. (Live not in idleness on the parents' inherited estates).

71. Lay not the whole faggot on the fire. (Live thriftily, spend not all at once).

72. Leap not from the chariot with your feet close together. (Do nothing inconsiderately).

73. Threaten not the stars. (Be not angry with your superiors).

74. Place not the candle against the wall. (Persist not in enlightening the stupid).

75. Write not in the snow. (Trust not your precepts to persons of an inconstant character).

Metaphysics and number theory.

According to Aristotle, number speculation is the most characteristic feature of Pythagoreanism. Things "are" number, or "resemble" number. To many Pythagoreans this concept meant that things are measurable and commensurable or proportional in terms of number--an idea of considerable significance for Western civilization. But there were also attempts to arrange a certain minimum number of pebbles so as to represent the shape of a thing--as, for instance, stars in a constellation that seem to represent an animal. For the Pythagoreans even abstracted things "have" their number: "justice" is associated with the number four and with a square, "marriage" with the number five, and so on. The psychological associations at work here have not been clarified.

For the Pythagoreans, the generation of number series was related to objects of Geometry as well as Cosmogony.

Monad (from Greek µ???? *monas*, "unit"; *monos*, "alone"), which according to the Pythagoreans, was a term for God or the first being, or the totality of all beings. Monad being the source or the One meaning without division.

The **Dyad** is a title used by the Pythagoreans for the number two, representing the principle of "twoness" or "otherness".

Numenius said that Pythagoras gave the name of Monad to God, and the name of Dyad to matter. Aristotle equated matter as the formation the elements (energies) into the material world as the static material was formed by the energies being acted upon by force or motion. Later Neoplatonic Philosophers and idealists like Plotinus treated the dyad as a second cause (Demiurge), which was the divine mind (Nous) that via a reflective nature (finiteness) causes matter to "appear" or become perceivable.

The **Triad** is a Pythagorean title for the number three. According to Priya Hemenway they considered it the most beautiful number, as it is the first number to equal the sum of all the terms below it, and the only number whose sum with those below equals the product of them and itself.

The **tetrad** or number is the first number formed by the addition and multiplication of equals. To the Pythagoreans, this symbol and number represented justice as it is the first number that is divisible every way into equal parts.

The **pentad** was a Pythagorean term for the number five. A pentagram, symbol of the pentad, was used by the Pythagoreans as a secret sign to recognize each other.In a passage from Lucian, he refers to the pentagram as the secret sign of brotherhood between the Pythagoreans. It represents the number five, life, power and invulnerability, which is why pentagrams are used as protection in banishing rites.

The **decad** was seen by the Pythagoreans as an "assembly point" and a symbol of earth and heaven. They regarded the decad as something perfect, which embraces the whole nature of number.

The harmony of the cosmos.

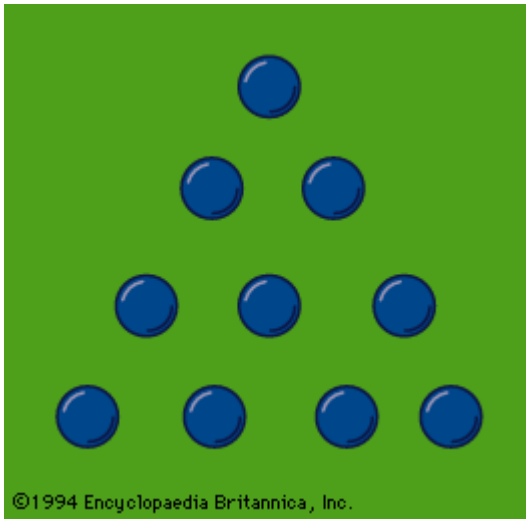


Figure 1: The Tetraktys (see text).

The sacred decad in particular has a cosmic significance in Pythagoreanism: its mystical name, *tetraktys* (meaning approximately "fourness"), implies 1 + 2 + 3 + 4 = 10; but it can also be thought of as a "perfect triangle," as in Figure 1. (see also cosmology)
Speculation on number and proportion led to an intuitive feeling of the *harmonia* ("fitting together") of the *kosmos* ("the beautiful order of things"); and the application of the *tetraktys* to the theory of music (see below *Music*) revealed a hidden order in the range of sound. Pythagoras may have referred, vaguely, to the "music of the heavens," which he alone seemed able to hear; and later Pythagoreans seem to have assumed that the distances of the heavenly bodies from the Earth somehow correspond to musical intervals--a theory that, under the influence of Platonic conceptions, resulted in the famous idea of the "harmony of the spheres." Though number to the early Pythagoreans was still a kind of cosmic matter, like the water or air proposed by the Ionians, their stress upon numerical proportions, harmony, and order comprised a decisive step toward a metaphysic in which form is the basic reality. (see also music theory)

The doctrine of opposites.
From the Ionians, the Pythagoreans adopted the idea of cosmic opposites, which they--perhaps secondarily--applied to their number speculation. The principal pair of opposites is the limit and the unlimited; the limit (or limiting), represented by the odd (3,5,7, . . .), is an active force effecting order, harmony, "cosmos," in the unlimited, represented by the even. All kinds of opposites somehow "fit together" within the cosmos, as they do, microcosmically, in an individual man and in the Pythagorean society. There was also a Pythagorean "table of ten opposites," to which Aristotle has referred--limit-unlimited, odd-even, one-many, right-left, male-female, rest-motion, straight-curved, light-darkness, good-evil, and square-oblong. The arrangement of this table reflects a dualistic conception, which was apparently not original with the school, however, or accepted by all of its members.
The Pythagorean number metaphysic was also reflected in its cosmology. The unit (1), being the starting point of the number series and its principle of construction, is not itself strictly a number; for, to be a number is to be even or odd, whereas, in the Pythagorean view, "one" is seen as *both*

even and odd. This ambivalence applies, similarly, to the total universe, conceived as the One. There was also a cosmogonical theory (of cosmic origins) that explained the generation of numbers and number-things from the limiting-odd and the unlimited-even--a theory that, by stages unknown to scholars, was ultimately incorporated into Plato's philosophy in his doctrine of the derivation of sensed realities from mathematical principles.

Mathematics and science.
Pythagorean thought was scientific as well as metaphysical and included specific developments in arithmetic and geometry, in the science of musical tones and harmonies, and in astronomy.

Arithmetic.
Early Pythagorean achievements in mathematics are unclear and largely disputable, and the following is, therefore, a compromise between the widely divergent views of modern scholars.

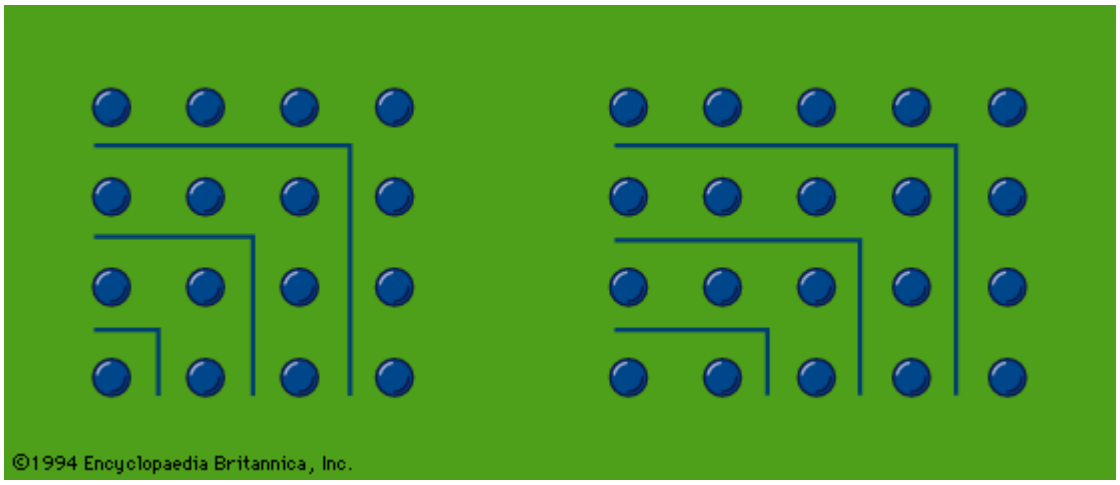


Figure 2: Gnomons of Pythagorean number theory (see text).

In the speculation on odd and even numbers, the early Pythagoreans used so-called *gnomones* (Greek: "carpenter's squares"). Judging from Aristotle's account, gnomon numbers, represented by dots or pebbles, were arranged in the manner shown in Figure 2. If a series of odd numbers is put around the unit as gnomons, they always produce squares; thus, the members of the series 4, 9, 16, 25, . . . are "square" numbers. If even numbers are depicted in a similar way, the resulting figures (which offer infinite variations) represent "oblong" numbers, such as those of the series 2, 6, 12, 20 . . . . On the other hand, a triangle represented by three dots (as in the upper part of the *tetraktys*) can be extended by a series of natural numbers to form the "triangular" numbers 6, 10 (the *tetraktys*), 15, 21. . . . This procedure, which was, so far, Pythagorean, led later, perhaps in the Platonic Academy, to a speculation on "polygonal" numbers.

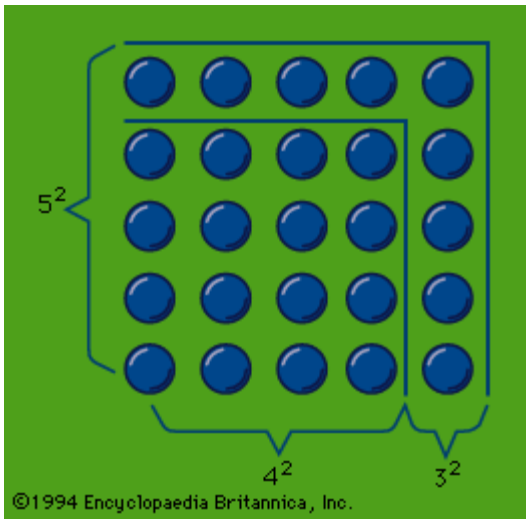


Figure 3: Gnomon for Pythagorean theorem. The marked off "carpenter's square"—comprising 3 groups of 3 dots each (3 × 3)—thus represents 3^2, which when added to 4^2 yields 5^2 (the total gnomon).

Probably the square numbers of the gnomons were early associated with the Pythagorean theorem (likely to have been used in practice in Greece, however, before Pythagoras), which holds that for a right triangle a square drawn on the hypotenuse is equal in area to the sum of the squares drawn on its sides; in the gnomons it can easily be seen, in the case of a 3,4,5-triangle for example (see Figure 3), that the addition of a square gnomon number to a square makes a new square: $3^2 + 4^2 = 5^2$, and this gives a method for finding two square numbers the sum of which is also a square.

Some 5th-century Pythagoreans seem to have been puzzled by apparent arithmetical anomalies: the mutual relationships of triangular and square numbers; the anomalous properties of the regular pentagon; the fact that the length of the diagonal of a square is incommensurable with its sides--*i.e.*, that no fraction composed of integers can express this ratio exactly (the resulting decimal is thus defined as irrational); and the irrationality of the mathematical proportions in musical scales. The discovery of such irrationality was disquieting because it had fatal consequences for the naive view that the universe is expressible in whole numbers; the Pythagorean Hippasus is said to have been expelled from the brotherhood, according to some sources even drowned, because he made a point of the irrationality.

In the 4th century, Pythagorizing mathematicians made a significant advance in the theory of irrational numbers, such as the-square-root-of-*n* ({sqroot *n*}), *n* being any rational number, when they developed a method for finding progressive approximations to {sqroot 2} by forming sets of so-called diagonal numbers.

Geometry.

In geometry, the Pythagoreans cannot be credited with any proofs in the Euclidean sense. They were evidently concerned, however, with some speculation on geometrical figures, as in the case of the Pythagorean theorem, and the concept that the point, line, triangle, and tetrahedron correspond to the elements of the *tetraktys*, since they are determined by one, two, three, and four points, respectively. They possibly knew practical methods of constructing the five regular

solids, but the theoretical basis for such constructions was given by non-Pythagoreans in the 4th century.
It is notable that the properties of the circle seem not to have interested the early Pythagoreans. But perhaps the tradition that Pythagoras himself discovered that the sum of the three angles of any triangle is equal to two right angles may be trusted. The idea of geometric proportions is probably Pythagorean in origin; but the so-called golden section--which divides a line at a point such that the smaller part is to the greater as the greater is to the whole--is hardly an early Pythagorean contribution. Some advance in geometry was made at a later date, by 4th-century Pythagoreans; *e.g.*, Archytas offered an interesting solution to the problem of the duplication of the cube--in which a cube twice the volume of a given cube is constructed--by an essentially geometrical construction in three dimensions; and the conception of geometry as a "flow" of points into lines, of lines into surfaces, and so on, may have been contributed by Archytas; but on the whole the numerous achievements of non-Pythagorean mathematicians were in fact more conspicuous than those of the Pythagoreans. (see also Pythagorean theorem)

Music.
The achievements of the early Pythagoreans in musical theory are somewhat less controversial. The scientific approach to music, in which musical intervals are expressed as numerical proportions, originated with them, as did also the more specific idea of harmonic "means." At an early date they discovered empirically that the basic intervals of Greek music include the elements of the *tetraktys*, since they have the proportions 1:2 (octave), 3:2 (fifth), and 4:3 (fourth). The discovery could have been made, for instance, in pipes or flutes or stringed instruments: the tone of a plucked string held at its middle is an octave higher than that of the whole string; the tone of a string held at the 2/3 point is a fifth higher; and that of one held at the 3/4 point is a fourth higher. Moreover, they noticed that the subtraction of intervals is accomplished by dividing these ratios by one another. In the course of the 5th century they calculated the intervals for the usual diatonic scale, the tone being represented by 9:8 (fifth minus fourth); *i.e.*, 3/2 {division} 4/3, and the semitone by 256:243 (fourth minus two tones); *i.e.*, 4/3 {division} (9/8 × 9/8). Archytas made some modification to this doctrine and also worked out the relationships of the notes in the chromatic (12-tone) scale and the enharmonic scale (involving such minute differences as that between A flat and G sharp, which on a piano are played by the same key).

Astronomy.
In their cosmological views the earliest Pythagoreans probably differed little from their Ionian predecessors. They made a point of studying the stellar heavens; but--with the possible exception of the theory of musical intervals in the cosmos--no new contributions to astronomy can be ascribed to them with any degree of probability. Late in the 5th century, or possibly in the 4th century, a Pythagorean boldly abandoned the geocentric view and posited a cosmological model in which the Earth, Sun, and stars circle about an (unseen) central fire--a view traditionally attributed to the 5th-century Pythagorean Philolaus of Croton.

HISTORY OF PYTHAGOREANISM

The life of Pythagoras and the origins of Pythagoreanism appear only dimly through a thick veil of legend and semihistorical tradition. The literary sources for the teachings of the Pythagoreans present extremely complicated problems. Special difficulties arise from the oral and esoteric

transmission of the early doctrines, the profuse accumulation of tendentious legends, and the considerable amount of confusion that was caused by the split in the school in the 5th century BC. In the 4th century, Plato's inclination toward Pythagoreanism created a tendency--manifest already in the middle of the century in the works of his pupils--to interpret Platonic concepts as originally Pythagorean. But the radical skepticism as to the reliability of the sources shown by some modern scholars has on the whole been abandoned in recent research. It now seems possible to extract bits of reliable evidence from a wide range of ancient authors, such as Porphyry and Iamblichus (see below *Neo-Pythagoreanism*).
Most of these literary sources hark back ultimately to the environment of Plato and Aristotle; and here the importance of one of Aristotle's students has become obvious, viz., the musicologist and philosopher Aristoxenus, who in spite of his bias possessed firsthand information independent of the point of view of Plato's Academy. The role played by Dicaearchus, another of Aristotle's pupils, and by the Sicilian historian Timaeus, of the early 3rd century BC, is less clear. Recently, the reliability of Aristotle's account of Pythagoreanism has also been emphasized against the doubts that had been expressed by some modern scholars; but Aristotle's sources, in turn, hardly lead farther back than to the late 5th century (perhaps to Philolaus; see below *Two Pythagorean sects*). In addition, there are scattered hints in various early authors and in some not very substantial remains of 4th-century Pythagorean literature. The mosaic of reconstruction thus has to be to some extent subjective.

2.6.3.1 Early Pythagoreanism.

Within the ancient Pythagorean movement four chief periods can be distinguished: early Pythagoreanism, dating from the late 6th century BC and extending to about 400 BC; 4th-century Pythagoreanism; the Hellenistic trends; and Neo-Pythagoreanism, a revival that occurred in the mid-1st century AD and lasted for two and a half centuries.

2.6.3.1.1 *Background.*

The background of Pythagoreanism is complex, but two main groups of sources can be distinguished. The Ionian philosophers (Thales, Anaximander, Anaximenes, and others) provided Pythagoras with the problem of a single cosmic principle, the doctrine of opposites, and whatever reflections of Oriental mathematics there are in Pythagoreanism; and from the technicians of his birthplace, the Isle of Samos, he learned to understand the importance of number, measurements, and proportions. Popular cults and beliefs current in the 6th century and reflected in the tenets of Orphism introduced him to the notions of occultism and ritualism and to the doctrine of individual immortality. In view of the shamanistic traits of Pythagoreanism, reminiscent of Thracian cults, it is interesting to note that Pythagoras seems to have had a Thracian slave. (see also opposites, table of, number system)

2.6.3.1.2 *Pythagorean communities.*

The school apparently founded by Pythagoras at Croton in southern Italy seems to have been primarily a religious brotherhood centred around Pythagoras and the cults of Apollo and of the Muses, ancient patron goddesses of poetry and culture. It became perhaps successively institutionalized and received different classes of esoteric members and exoteric sympathizers. The rigorism of the ritual and ethical observances demanded of the members is unparalleled in early Greece; in addition to the rules of life mentioned above, it is fairly well attested that secrecy and a long silence during the novitiate were required. The exoteric associates, however,

were politically active and established a Crotonian hegemony in southern Italy. About 500 BC a coup by a rival party caused Pythagoras to take refuge in Metapontum, where he died. During the early 5th century, Pythagorean communities, inspired by the original school at Croton, existed in many southern Italian cities, a fact that led to some doctrinal differentiation and diffusion. In the course of time the politics of the Pythagorean parties became decidedly antidemocratic. About the middle of the century a violent democratic revolution swept over southern Italy; in its wake, many Pythagoreans were killed, and only a few escaped, among them Lysis of Tarentum and Philolaus of Croton, who went to Greece and formed small Pythagorean circles in Thebes and Phlious.

2.6.3.1.3 ***Two Pythagorean sects.***

Little is known about Pythagorean activity during the latter part of the 5th century. The differentiation of the school into two main sects, later called *akousmatikoi* (Greek: *akousma,* "something heard," viz., the esoteric teachings) and *mathematikoi* (Greek: *mathematikos,* "scientific"), may have occurred at that time. The acousmatics devoted themselves to the observance of rituals and rules and to the interpretation of the sayings of the master; the "mathematics" were concerned with the scientific aspects of Pythagoreanism. Philolaus, who was rather a mathematic, probably published a summary of Pythagorean philosophy and science in the late 5th century.

2.6.3.2 4th-century Pythagoreanism.

In the first half of the 4th century, Tarentum, in southern Italy, rose into considerable significance. Under the political and spiritual leadership of the mathematic Archytas, a friend of Plato, Tarentum became a new centre of Pythagoreanism, from which acousmatics--so-called Pythagorists who did not sympathize with Archytas--went out travelling as mendicant ascetics all around the Greek-speaking world. The acousmatics seem to have preserved some early Pythagorean *Hieroi Logoi* and ritual practices. Archytas himself, on the other hand, concentrated on scientific problems, and the organization of his Pythagorean brotherhood was evidently less rigorous than that of the early school. After the 380s there was a give-and-take between the school of Archytas and the Academy of Plato, a relationship that makes it almost impossible to disentangle the original achievements of Archytas from joint involvements (but see above, *Geometry* and *Music*).

2.6.3.3 The Hellenistic Age.

Whereas the school of Archytas apparently sank into inactivity after the death of its founder (probably after 350 BC), the Academics of the next generation continued "Pythagorizing" Platonic doctrines, such as that of the supreme One, the indefinite dyad (a metaphysical principle), and the tripartite soul. At the same time, various Peripatetics of the school of Aristotle, including Aristoxenus, collected Pythagorean legends and applied contemporary ethical notions to them. In the Hellenistic Age, the Academic and Peripatetic views gave rise to a rather fanciful antiquarian literature on Pythagoreanism. There also appeared a large and yet more heterogeneous mass of apocryphal writings falsely attributed to different Pythagoreans, as if attempts were being made to revive the school. The texts fathered on Archytas display Academic and Peripatetic philosophies mixed with some notions that were originally Pythagorean. Other texts were fathered on Pythagoras himself or on his immediate pupils, imagined or real. Some show, for instance, that Pythagoreanism had become confused with Orphism; others suggest that Pythagoras was considered a magician and an astrologist; there are

also indications of Pythagoras "the athlete" and "the Dorian nationalist." But the anonymous authors of this pseudo-Pythagorean literature did not succeed in reestablishing the school, and the "Pythagorean" congregations formed in early imperial Rome seem to have had little in common with the original school of Pythagoreanism established in the late 6th century BC; they were ritualistic sects that adopted, eclectically, various occult practices.

2.6.3.4 Neo-Pythagoreanism.

With the ascetic sage Apollonius of Tyana, about the middle of the 1st century AD, a distinct Neo-Pythagorean trend appeared. Apollonius studied the Pythagorean legends of the previous centuries, created and propagated the ideal of a Pythagorean life--of occult wisdom, purity, universal tolerance, and approximation to the divine--and felt himself to be a reincarnation of Pythagoras. Through the activities of Neo-Pythagorean Platonists, such as Moderatus of Gades, a pagan trinitarian, and the arithmetician Nicomachus of Gerasa, both of the 1st century AD, and, in the 2nd or 3rd century, Numenius of Apamea, forerunner of Plotinus (an epoch-making elaborator of Platonism), Neo-Pythagoreanism gradually became a part of the expression of Platonism known as Neoplatonism; and it did so without having achieved a scholastic system of its own. The founder of a Syrian school of Neoplatonism, Iamblichus, a pupil of Porphyry (who in turn had been a pupil of Plotinus), thought of himself as a Pythagorean sage and about AD 300 wrote the last great synthesis of Pythagoreanism, in which most of the disparate post-classical traditions are reflected. It is characteristic of the Neo-Pythagoreans that they were chiefly interested in the Pythagorean way of life and in the pseudoscience of number mysticism. On a more popular level, Pythagoras and Archytas were remembered as magicians. Moreover, it has been suggested that Pythagorean legends were also influential in guiding the Christian monastic tradition.

2.6.3.5 Medieval and modern trends.

In the Middle Ages the popular conception of Pythagoras the magician was combined with that of Pythagoras "the father of the quadrivium"; *i.e.*, of the more specialized liberal arts of the curriculum. From the Italian Renaissance onward, some "Pythagorean" ideas, such as the tetrad, the golden section, and harmonic proportions, became applied to aesthetics. To many Humanists, moreover, Pythagoras was the father of the exact sciences. In the early 16th century, Nicolaus Copernicus, who developed the view that the Earth revolves around the Sun, considered his system to be essentially Pythagorean or "Philolaic," and Galileo was called a Pythagorean. The 17th-century Rationalist G.W. Leibniz appears to have been the last great philosopher and scientist who felt himself to be in the Pythagorean tradition. (see also humanism)

It is doubtful whether advanced modern philosophy has ever drawn from sources thought to be distinctly Pythagorean. Yet Platonic-Neoplatonic notions, such as the mathematical conception of reality or the philosopher's union with the universe and various mystical beliefs are still likely to be stamped as Pythagorean in origin. Even today a relatively uncritical admiration of Pythagoreanism is common. (see also Platonism)

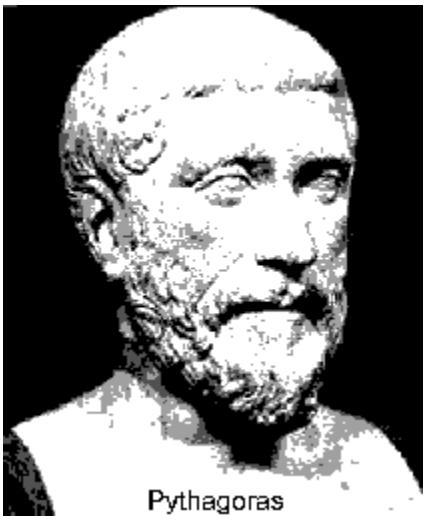
2.6.4 EVALUATION

The history of the projection of Pythagoreanism into subsequent thought indicates how fertile some of its core concepts were. Plato is here the great catalyst; but it is possible to perceive behind him, however dimly, a series of Pythagorean ideas of paramount potential significance: the combination of religious esoterism (or exclusivism) with the germs of a new philosophy of

mind, present in the belief in the progress of the soul toward the actualization of its divine nature and toward knowledge; stress upon harmony and order, and upon limit as the good; the primacy of form, proportion, and numerical expression; and in ethics, and emphasis upon such virtues as friendship and modesty. The fact that Pythagoras, to later ages, also became alternatively conceived of as a Dorian nationalist, a sportsman, an educator of the people, or a great magician is a more curious consequence of the productivity of his teaching.

Pythagoras and the Pythagoreans

Historically, **Pythagoras** means much more that the familiar theorem about right triangles. The philosophy of Pythagoras and his school has impacted the very fiber of mathematics and physics, even the western tradition of liberal education no matter what the discipline.



Pythagorean philosophy was the prime source of inspiration for Plato and Aristotle; the influence of these philosophers is without question and is immeasurable.

Pythagoras and the Pythagoreans

Little is known of his life. Pythagoras (fl 580-500, BC) was born in Samos on the western coast of what is now Turkey. He was reportedly the son of a substantial citizen, Mnesarchos. There he lived for many years under the rule of the tyrant Polycrates, who had a tendency to switch alliances in times of conflict -- which were frequent.
He met Thales, likely as a young man, who recommended he travel to Egypt. It seems certain that he gained much of his knowledge from the Egyptians, as had Thales before him.
Probably because of continual conflicts and strife in Samos, Pythagoras settled in Croton, on the eastern coast of Italy, a place of relative peace and safety.
Even so, just as he arrived, Croton lost a war to neighboring city Locri, but soon thereafter defeated utterly the luxurious city of Sybaris.
This is where Pythagoras began his society.

The Pythagorean School

★The school of Pythagoras was every bit as much a religion as a school of mathematics. For example, here are some of the rules:

- To abstain from beans.
- Not to pick up what has fallen.
- Not to touch a white cock.
- Not to stir the fire with iron.
- ⋮
- Do not look in a mirror beside a light.

Vegetarianism was strictly practiced probably because Pythagoras preached the transmigration of souls.
The school of Pythagoras represents the **mystic tradition** in contrast with the scientific!!
Indeed, Pythagoras regarded himself as a mystic and even *semi-divine*. Said Pythagoras
"There are men, gods, and men like Pythagoras."
It is likely that Pythagoras was a charasmatic.
Life in the Pythagorean society was more-or-less egalitarian.

- The Pythagorean school regarded men and women equally.
- They enjoyed a common way of life.
- Property was communal.
- Even mathematical discoveries were communal and by association attributed to Pythagoras himself -- even from the grave. Hence, exactly what Pythagoras discovered personally is difficult to ascertain.

The Pythagorean Philosophy

The basis of the Pythagorean philosophy is simply stated:
"There are three kinds of men and three sorts of people that attend the Olympic Games. The lowest class is made up of those who come to buy and sell, the next above them are those who compete. Best of all, however, are those who come simply to look on. The greatest purification of all is, therefore, disinterested science, and it is the man who devotes himself to that, the true philosopher, who has most effectually released himself from the 'wheel of birth.'"
The message of this passage is radically in conflict with modern values. We need only consider sports and politics.
★Is not reverence these days is bestowed only on the ``super-stars''?
★Are not there ubiquitous demands for *accountability*!!

The Pythagorean Philosophy

The *gentleman*, of this passage, has had a long run with this philosophy, because he was associated with the Greek genius, because the ``virtue of comtemplation'' acquired theological endorsement, and because the ideal of disinterested truth dignified the academic life.

The Pythagorean Philosophy ála Bertrand Russell

From Bertrand Russell, we have
"It is to this gentleman that we owe pure mathematics. The comtemplative ideal -- since it led to pure mathematics -- was the source of a useful activity. This increased it's prestige and gave it a success in theology, in ethics, and in philosophy."
Mathematics, so honored, became the model for other sciences. *Thought* became superior to the senses; *intuition* became superior to observation.
The combination of mathematics and theology began with Pythagoras. It characterized the religious philosophy in Greece, in the Middle ages, and down through Kant. In Plato, Aquinas, Descartes, Spinoza and Kant there is a blending of religion and reason, of moral aspiration with logical admiration of what is timeless.
Platonism was essentially Pythagorean ism. The whole concept of an eternal world revealed to intellect but not to the senses can be attributed from the teachings of Pythagoras.
The Pythagorean School gained considerable influence in Croton and became politically active -- on the side of the aristocrasy. Probably because of this, after a time the citizens turned against him and his followers, burning his house. Forced out, he moved to **Metapontum**, also in Southern Italy. Here he died at the ages of eighty. His school lived on, alternating between decline and re-emergence, for several hundred years.
Tradition holds that Pythagoras left no written works, but that his ideas were carried on by his eager disciples.
What is known of the Pythagorean school is from a book written by the Pythagorean, **Philolaus** of Tarentum. From this book Plato learned the philosophy of Pythagoras.
The dictum of the Pythagorean school was *All is number*.
What this meant was that all things of the universe had a numerical attribute that uniquely described them. For example,

- The number **one** : the number of reason.
- The number **two**: the first even or female number, the number of opinion.
- The number **three**: the first true male number, the number of harmony.
- The number **four**: the number of justice or retribution.
- The number **five**: marriage.
- The number **six**: creation
- ⋮
- The number **ten**: the *tetractys*, the number of the universe.

Pythagorean Mathematics

One point: generator of dimensions.
Two points: generator of a line of dimension one
Three points: generator of a triangle of dimension two
Four points: generator of a tetrahedron, of dimension three.

The sum of these is ten and represents all dimensions. Note the abstraction of concept. This is a distance from ``fingers and toes''.

Pythagorean Mathematics

Classification of numbers. The distinction between even and odd numbers certainly dates to Pythagoras. From Philolaus, we learn that
"...number is of two special kinds, odd and even, with a third, even-odd, arising from a mixture of the two; and of each kind there are many forms."
And these, even and odd, correspond to the usual definitions. But *even-odd* means a product of an even and odd number. Note: orginally the number 2 was not considered even.

Prime or **incomposite** numbers and **secondary** or **composite** numbers are defined in Philolaus:

- A **prime** number is rectilinear, meaning that it can only be set out in one dimension. The number 2 was not originally regarded as a prime number, or even as a number at all.
- A **composite** number is that which is measured by some number. (Euclid)
- Two numbers are **prime to one another** or **composite to one another** if their greatest common divisor is one or greater than one, respectively.

Proposition. There are an infinite number of primes.
Proof. (Euclid) Suppose that there exist only finitely many primes $p_1 < p_2 < ... < p_r$. Let $N = (p_1)(p_2)...(p_r) > 2$. The integer *N*-1, being a product of primes, has a prime divisor p_i in common with *N*; so, p_i divides *N*- (*N*-1) =1, which is absurd!
The search for primes goes on. **Eratsothenes** (276 B.C. - 197 B.C.), who worked in Alexandria, devised a *seive* for determining primes. This seive is based on a simple concept:
Lay off all the numbers, then mark of all the multiples of 2, then 3, then 5, and so on. A prime is determined when a number is not marked out. So, 3 is uncovered after the multiples of two are marked out; 5 is uncovered after the multiples of two and three are marked out.

The Primal Challenge
The search for large primes goes on: Below is a list of the largest found to date. For a great deal of information on primes, including the numbers below, check out the The Primes Home Page A special method, the *Lucas-Lehmer* test has been derived to check primality.

```
2^859433-1 (258,716 digits); Slowinski and Gage, 1994
2^756839-1 (227832 digits); Slowinski and Gage, 1992
391581*2^216193-1 (65087 digits); Noll and others, 1989
2^216091-1 (65050 digits); Slowinski, 1985
3*2^157169+1 (47314 digits); Jeffrey Young, 1995
9*2^149143+1 (44898 digits); Jeffrey Young, 1995
9*2^147073+1 (44275 digits); Jeffrey Young, 1995
9*2^145247+1 (43725 digits); Jeffrey Young, 1995
2^132049-1 (39751 digits); Slowinski, 1983
9*2^127003+1 (38233 digits); Jeffrey Young, 1995
```

Subdivisions of even numbers are reported by Nicomachus (a neo-Pythagorean, ~100 A.D.).

★ even-even -- 2^n
★ even-odd -- 2(2*m*+1)
★ odd-even -- $2^{n+1}(2m+1)$

Similar subdivisions of odd numbers are:

★ prime and incomposite -- ordinary primes excluding 2,

★ secondary and composite -- ordinary composite with prime factors only,
★ relatively prime -- two composite numbers but prime and incomposite to another number, e.g. 9 and 25.

Actually the third category is wholy subsumed by the second.
Also ascribed to the Pythagorean s is the study of perfect and amicable and deficient numbers.
A number n is **perfect** if the sum of its divisors is itself: Examples: (6, 28, 496, 8128, ...) In Euclid, we find the proposition: *If* $2^p - 1$ *is prime, then* $(2^p - 1)2^{p-1}$ *is perfect*. (Try , p= 2, 3, 5, 7 to get the numbers above.)
The pair of numbers *a* and *b* are called **Amicable** If the divisors of *a* sum to *b* and if the divisors of *b* sum to *a*. Example: 220 and 284.
In addition, the number *a* was classified as **abundant** or **deficient** according as their divisors summed greater or less than *a*, respectively.
Example: 12-divisors are: 6,4,3,2,1-- $\sum = 16$. So, 12 is abundant.
Example: All primes are deficient.

Mersenne Primes

Just the fact of finding perfect numbers using the previous propositions has spawned a cottage industry of determining those numbers *p* for which $2^p - 1$ is prime. Such primes are called **Mersenne** (1588-1648) primes after the friar of the 17 th century So far 33 have been found, though it is unknown if there is another between the 32nd and 33rd. It's not known if there are an infinity.
Recall,

$$\begin{aligned} 2^2 - 1 &= 3 \\ 2^3 - 1 &= 7 \\ 2^5 - 1 &= 31 \\ 2^7 - 1 &= 127 \\ 2^13 - 1 &= 8191 \end{aligned}$$

Mersenne Primes

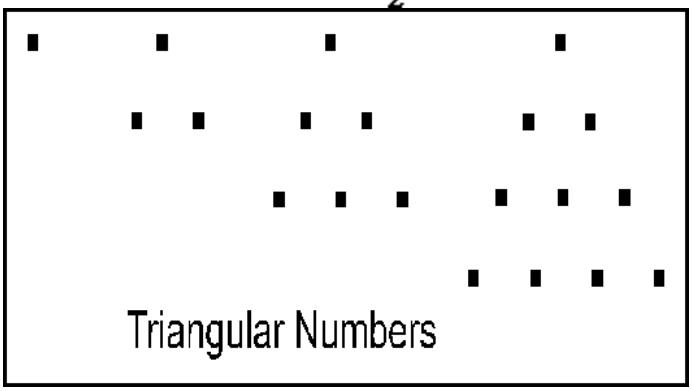
number	p	year	by
1-5	2,3,5,7,13	in or before the middle ages	
6-7	17,19	1588	Cataldi
8	31	1750	Euler
9	61	1883	Pervouchine
10	89	1911	Powers
11	107	1914	Powers
12	127	1876	Lucas
13-14	521,607	1952	Robinson
15-17	1279,2203,2281	1952	Lehmer
18	3217	1957	Riesel
19-20	4253,4423	1961	Hurwitz & Selfridge
21-23	9689,9941,11213	1963	Gillies
24	19937	1971	Tuckerman
25	21701	1978	Noll & Nickel
26	23209	1979	Noll
27	44497	1979	Slowinski & Nelson

28	86243	1982	Slowinski
29	110503	1988	Colquitt & Welsh jr.
30	132049	1983	Slowinski
31	216091	1985	Slowinski
32?	756839	1992	Slowinski & Gage
33?	859433	1994	Slowinski and Gage,

Figurate Numbers. Numbers geometrically constructed had a particular importance to the Pythagorean s.

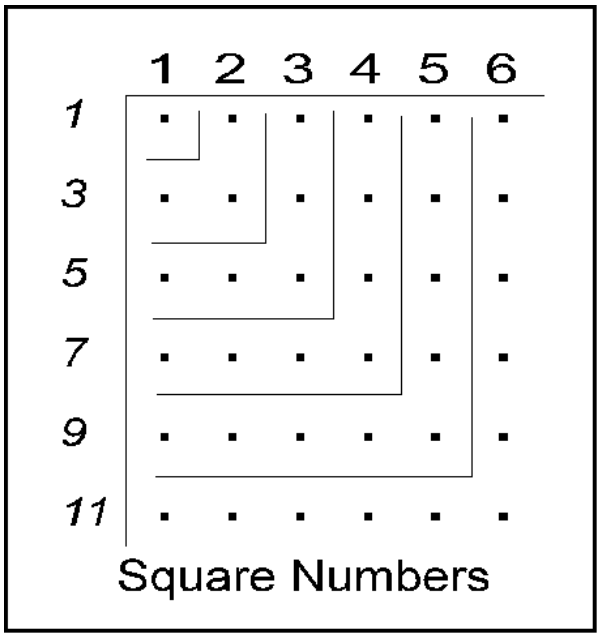
Triangular numbers. These numbers are 1, 3, 6, 10, ... The general form is the familiar

$$1+2+3+\ldots+n=\frac{n(n+1)}{2}.$$



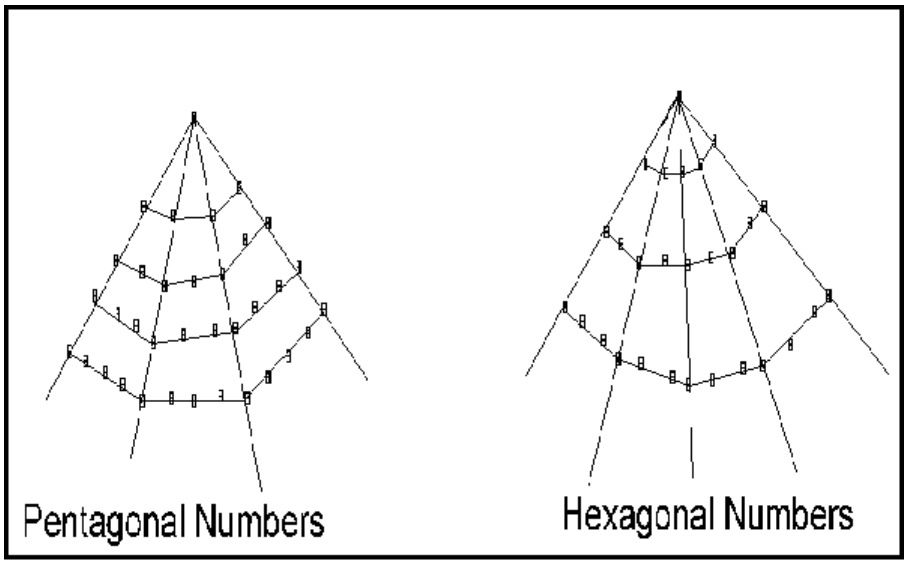
Square numbers These numbers are clearly the squares of the integers 1, 4, 9, 16, and so on. Represented by a square of dots, they prove(?) the well known formula

$$1+3+5+\ldots+(2n-1)=n^2.$$



The *gnomon* is basically an architect's template that marks off "similar" shapes.
Note the *gnomon* has been placed so that at each step, the next odd number of dots is placed.
Figurate Numbers of any kind can be calculated.

The **Pentagonal** and **Hexagonal** numbers are shown in the graphs below.



Note that the sequences have sums given by

$$1+4+7+\ldots+(3k-2)=\frac{3}{2}n^2-\frac{1}{2}n$$

and

$$1+5+9+\ldots+(4n-3)=2n^2-n.$$

Similarly, polygonal numbers of all orders are dsignated; this process can be extended to three dimensional space, where there results the **polyhedral numbers**. Philolaus is reported to have said:
All things which can be known have number; for it is not possible that without number anything can be either conceived or known.

The Pythagorean Theorem

Whether Pythagoras learned about the 3, 4, 5 right triangle while he studied in Egypt or not, he was certainly aware of it. This fact though could not but strengthen his conviction that *all is number*. It would also have led to his attempt to find other forms. How might he have done this? One place to start would be with the square numbers, and arrange that three consecutive numbers be a Pythagorean triple! Consider for any odd number *m*,

$$m^2+(\frac{m^2-1}{2})^2=(\frac{m^2+1}{2})^2,$$

which is the same as

$$m^2+\frac{m^4}{4}-\frac{m^2}{2}+\frac{1}{4}=\frac{m^4}{4}+\frac{m^2}{2}+\frac{1}{4},$$

or

$$m^2 = m^2.$$

Put the gnomon around n^2. The next number is $2n+1$, which we suppose to be a square.

$$2n + 1 = m^2,$$

which implies

$$n = \frac{1}{2}(m^2 - 1),$$

and therefore

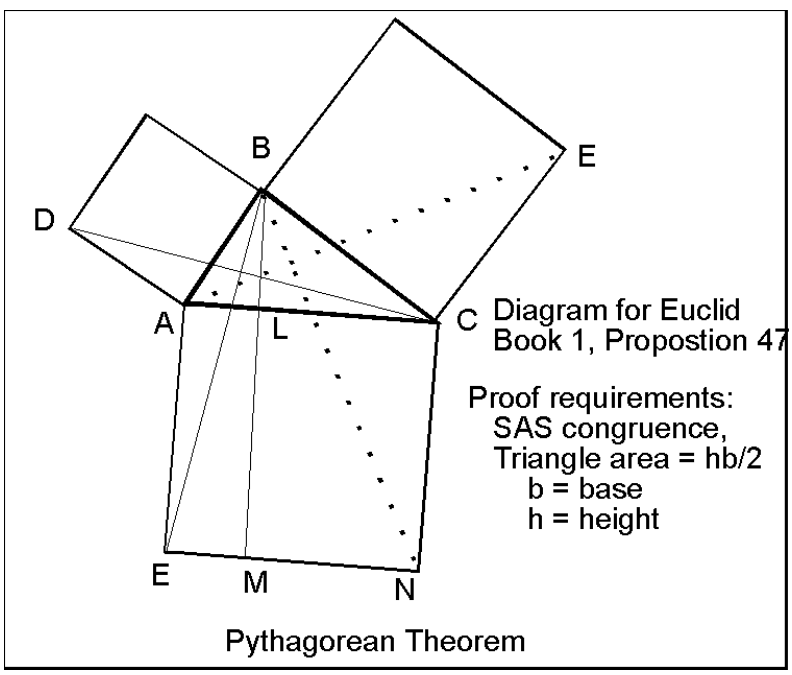
$$n + 1 = \frac{1}{2}(m^2 + 1).$$

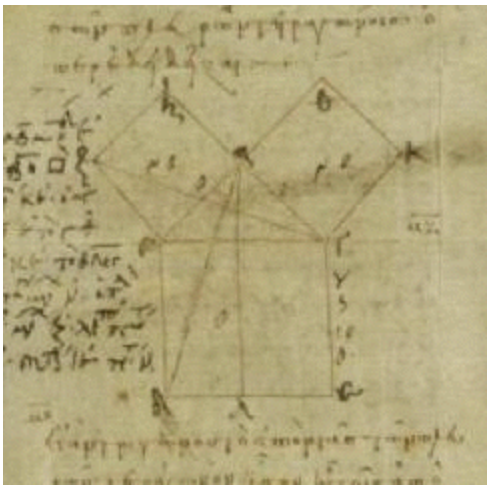
It follows that

$$m^2 + \frac{m^4}{4} - \frac{m^2}{2} + \frac{1}{4} = \frac{m^4}{4} + \frac{m^2}{2} + \frac{1}{4}$$

This idea evolved over the years and took other forms.
Did Pythagoras or the Pythagorean s actually prove the Pythagoras Theorem? Probably not. Although a proof is simple to give, the Pythagorean s had only a limited theory of similarity. And perhaps the reason was that rigor had not yet advanced to that level, at least in the early and middle period. The late Pythagorean s (400 B.C) however probably did supply a proof of this most famous of theorems.

Theorem I-47. In right-angled triangles, the square upon the hypotenuse is equal to the sum of the squares upon the legs.





This figure is modelled after the original figure used by Euclid to prove the result. It is known to the French as *pon asinorum* and to the Arabs as the *Figure of the Bride*.

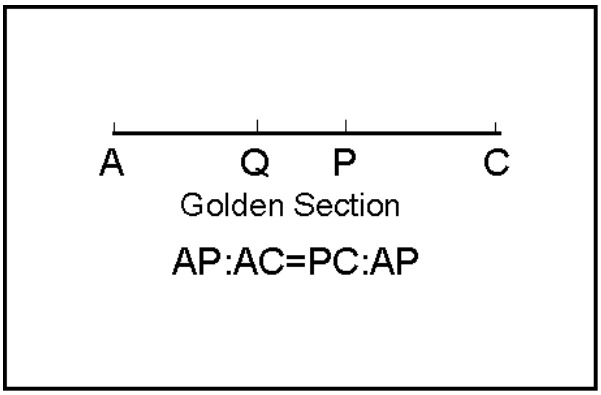
More Pythagorean Geometry

Contributions by the Pythagorean s include

- Various theorems about triangles, parallel lines, polygons, circles, spheres and regular polyhedra.
- Work on a class of problems in the applications of areas. (e.g. to construct a polygon of given area and similar to another polygon.)
- Given a line segment, construct on part of it or on the line segment extended a parallelogram equal to a given rectilinear figure in area and falling short or exceeding by a parallelogram similar to a given one. (In modern terms, solve $\frac{b}{c}x^2 + ax = d$.)

From Kepler we have the quote
``Geometry has two great treasures: one is the Theorem of Pythagoras; the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel."

A line AC divided into **extreme and mean ratio** is defined to mean that it is divided into two parts, AP and PC so that AP:AC=PC:AP, where AP is the longer part.



Let *AP*=*x* and *AC*= *a*. Then the golden section is

$$\frac{x}{a} = \frac{a-x}{x},$$

and this gives the quadratic equation

$$x^2 + ax - a^2.$$

The solution is

$$x = \frac{-1 \pm \sqrt{5}}{2} a.$$

The **golden section** is the positive root:

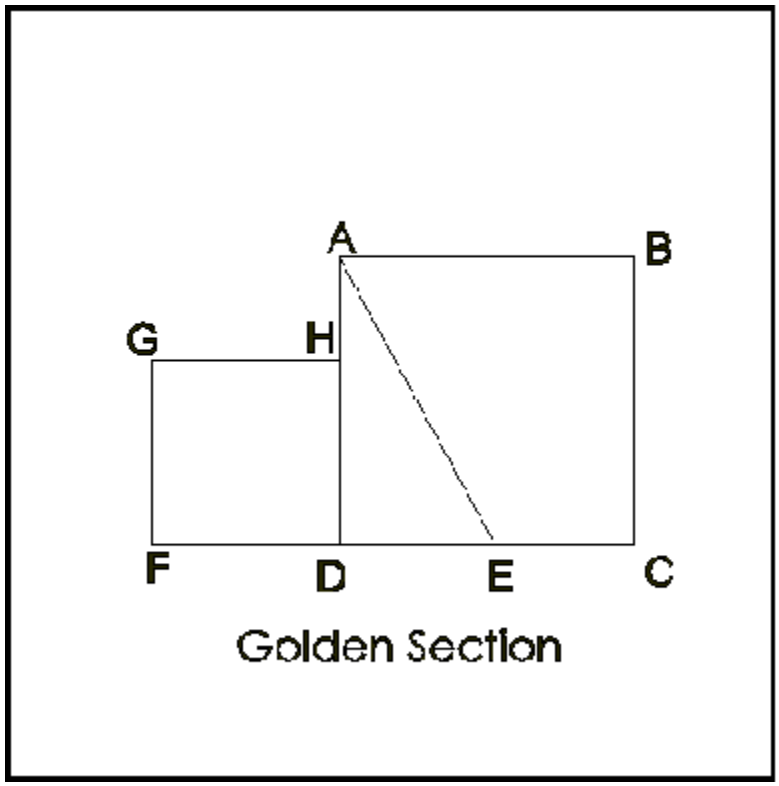
$$x = \frac{\sqrt{5}-1}{2}$$

And this was all connected with the construction of a pentagon.

The Pythagorean Pentagram

First we need to construct the golden section. The geometric construction, the only kind accepted, is illustrated below.

Assume the square ABCE has side length *a*. Bisecting DC at E construct the diagonal AE, and extend the segment ED to EF, so that EF=AE. Construct the square DFGH. The line AHD is divided into extrema and mean ratio.



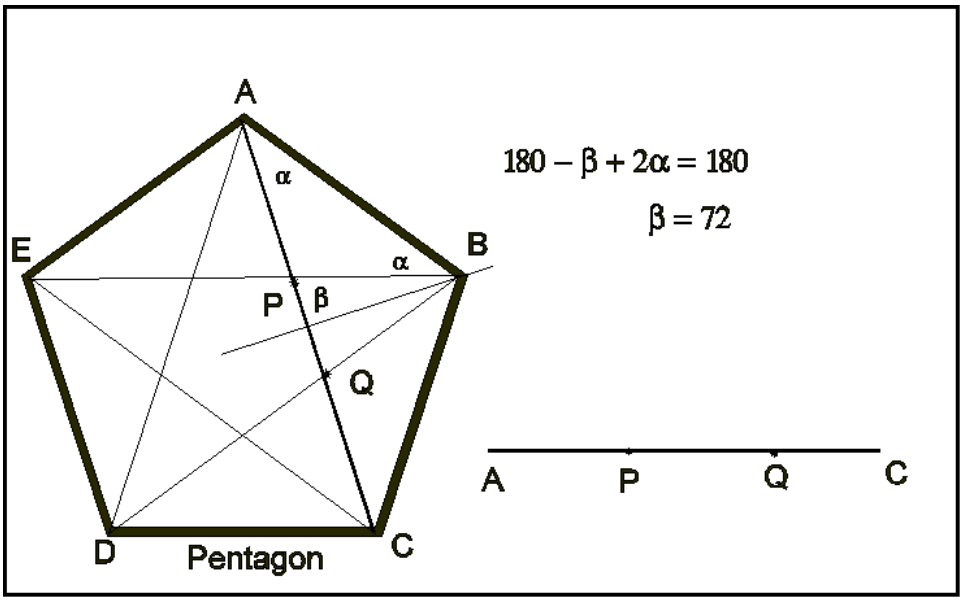
Verification. Note,

$$|AE|^2 = |AD|^2 + |DE|^2 = a^2 + (a/2)^2 = \frac{5}{4}a.$$

Thus,

$$|DH| = (\frac{\sqrt{5}}{2} - \frac{1}{2})a = \frac{\sqrt{5}-1}{2}\,a.$$

The key to the compass and ruler construction of the pentagon is the construction of the isoceles triangle with angles $36^o,\ 72^o,$ and 72°. We begin this construction from the line AC. Note the figure



Divide a line AC into the `section' with respect to both endpoints. So PC:AC=AP:PC; also AQ:AC=QC:AQ. Bisect the line AC and construct on the perpendicular at this midpoint the point B so that AP=PB=QB=QC.

Define $\alpha :=$ ∠PAB and $\beta :=$ ∠QPB. Then $180^o - \beta - 2\alpha = 180^o$. This implies $\alpha = \frac{1}{2}\beta$, and hence $(2 + \frac{1}{2})\beta = 180$. Solving for β we, get $\beta = 72^o$. Since △PBQ is isoceles, the angle ∠QBP 32°. Now complete the line BE=AC and the line BD=AC and connect edges AE, ED and DC. Apply similarity of triangles to show that all edges have the same length. This completes the proof.

Regular Polygons

The only regular polygons known to the Greeks were the equilaterial triangle and the pentagon. It was not until about 1800 that C. F. Guass added to the list of constructable regular polyons by

showing that there are three more, of 17, 257, and 65,537 sides respectively. Precisely, he showed that the constructable regular polygons must have

$$2^m p_1 p_2 \cdots p_r$$

sides where the $p_1, \ldots, p_r$ are distinct **Fermat primes**. Recall, a Fermat prime is a prime having the form

$$2^{2^n} + 1.$$

Fermat (1630) conjectured that all numbers of this kind are prime.

Fermat

Pierre Fermat (1601-1665), was a court attorney in Toulouse (France). He was an avid mathematician and even participated in the fashion of the day which was to reconstruct the masterpieces of Greek mathematics. He generally refused to publish, but communicated his results by letter.

Regular Polygons

Are there any other Fermat primes? Here's what's known to date.

p	$2^{2^p} + 1$	Factors	Discoverer
0	3	3	ancient
1	5	5	ancient
2	17	17	ancient
3	257	257	ancient
4	65537	65537	ancient
5	4,294,957,297	641, 6,700,417	Euler, 1732
6	18446744073709551617	67280421310721, 274177	
7	88 digits	composite	
8	177 digits	composite	
9	354 digits	composite	Lenstra, et.al., 1990
10	709 digits	unknown	
11	1409 digits	composite	Brent and Morain, 1988

By the theorem of Gauss, there are constructions of regular polygons of only 3, 5 ,15 , 257, and 65537 sides, plus multiples,

$$2^m p_1 p_2 \cdots p_r$$

sides where the $p_1, \ldots, p_r$ are distinct **Fermat primes**.

The Pythagorean Theory of Proportion

Besides discovering the five regular solids, Pythagoras also discovered the theory of proportion. Pythagoras had probably learned in Babylon the three basic means, the **arithmetic**, the **geometric**, and the **subcontrary** (later to be called the **harmonic**).

Beginning with *a*>*b*>*c* and denoting *b* as the **--mean** of *a* and *c*, they are:

$$\frac{a-b}{b-c} = \frac{a}{a} \qquad arithmetic \quad a+c=2b$$

$$\frac{a-b}{b-c} = \frac{a}{b} \qquad geometric \quad ac=b^2$$

$$\frac{a-b}{b-c} = \frac{a}{c} \qquad subcontrary \quad \frac{1}{a}+\frac{1}{c}=\frac{2}{b}$$

The Pythagorean Theory of Proportion

In fact, Pythagoras or more probably the Pythagorean s added seven more proportions. Here are a few, given in modern notation:

$$\frac{a^2+c^2}{a+c} = b$$

$$a = b + c - \frac{c^2}{b}$$

$$c = a + b - \frac{a^2}{b}$$

$$c^2 = 2ac - b^2$$

$$a^2 + c^2 = a(b+c)$$

$$a = b + c$$

Note: each of these is expressible in the notation of proportion, as above.

Recall,

$$\frac{a-b}{b-c} = \frac{a}{a} \qquad arithmetic \quad a+c=2b$$

$$\frac{a-b}{b-c} = \frac{a}{b} \qquad geometric \quad ac=b^2$$

$$\frac{a-b}{b-c} = \frac{a}{c} \qquad harmonic \quad \frac{1}{a}+\frac{1}{c}=\frac{2}{b}$$

Allowing *A* , *G*, and *H* denote the arithmetic, geometric and harmonic means, the Pythagorean s called the proportion

$$A : G = G : H$$

the **perfect** proportion. The proportion

$$a : A = H : c$$

was called the **musical** proportion.

The Discovery of Incommensurables

This discovery is usually given to **Hippasus of Metapontum** (5^{th}cent B.C.). One account gives that the Pythagorean s were at sea at the time and when Hippasus produced an element which denied virtually all of Pythagorean doctrine, he was thrown overboard.

Theorem. $\sqrt{2}$ is incommensurable with 1.

Proof. Suppose that $\sqrt{2} = \frac{a}{b}$, with no common factors. Then

$$2 = \frac{a^2}{b^2}$$

or

$$a^2 = 2b^2.$$

Thus $2 \mid a^2$, and hence $2 \mid a$. So, *a*=2*c* and it follows that

$$2c^2 = b^2,$$

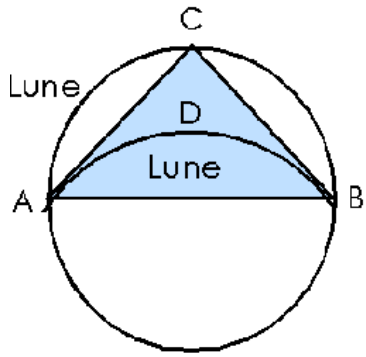
whence by the same reasoning yields that $2 \mid b$. This is a contradiction. width .1in height .1in depth 0pt

But is this the actual proof known to the Pythagorean s? Note: Unlike the Babylonnians or Egyptians, the Pythagorean s recognized that this class of numbers was wholly different from the rationals.

``Properly speaking, we may date the very beginnings of ``theoretical'' mathematics to the first proof of irrationality, for in ``practical'' (or applied) mathematics there can exist no irrational numbers.'' Here a problem arose that is analogous to the one whose solution initiated theoretical natural science: it was necessary to ascertain something that it was absolutely impossible to observe (in this case, the incommensurability of a square's diagonal with its side). The discovery of incommensurability was attended by the introduction of indirect proof and, apparently in this connection, by the development of the definitional system of mathematics. In general, the proof of irrationality promoted a stricter approach to geometry, for it showed that the evident and the trustworthy do not necessarily coincide.

Other Pythagorean Geometry

Quadrature of certain **lunes** was performed by **Hippocrates of Chios**. He is also credited with the arrangement of theorems in an order so that one may be proved from a previous one (as we see in Euclid).



The large lune *ABD* is similar to the smaller lune (with base on one leg of the right isosceles triangle $\triangle ABC$. **Proposition:**The area of the large lune *ABD* is the area of the semi-circle less the area of the triangle $\triangle ABC$.

Tetractys

Using a The **Tetractys**, also known as the **decad**, is a triangular figure consisting of ten points arranged in four rows: one, two, three, and four points in each row. As a mystical symbol, it was very important to the followers of the secret worship of the Pythagoreans.

```
      *
     * *
    * * *
   * * * *
```

Tetractys - Pythagorean symbol

1. The Tetractys symbolized the four elements - earth, air, fire, and water.
2. The first four numbers also symbolized the harmony of the spheres and the Cosmos
3. The first four numbers added up to ten, which was unity of a higher order
4. The Tetractys represented the organization of space:
    1. the first row represented zero-dimensions (a point)
    2. the second row represented one-dimension (a line of two points)
    3. the third row represented two-dimensions (a plane defined by a triangle of three points)
    4. the fourth row represented three-dimensions (a triangular pyramid defined by four points)

A prayer of the Pythagoreans shows the importance of the Tetractys (sometimes called the "Mystic Tetrad"), as the prayer was addressed to it.

"Bless us, divine number, thou who generated gods and men! O holy, holy Tetractys, thou that containest the root and source of the eternally flowing creation! For the divine number begins with the profound, pure unity until it comes to the holy four; then it begets the mother of all, the all-comprising, all-bounding, the first-born, the never-swerving, the never-tiring holy ten, the keyholder of all".

As a portion of the secret religion, initiates were required to swear a secret oath by the Tetractys. They then served as novices for a period of silence lasting three years.

The Pythagorean oath also mentioned the Tetractys:

"By that pure, holy, four lettered name on high,
nature's eternal fountain and supply,
the parent of all souls that living be,
by him, with faith find oath, I swear to thee."

The Pythagorean SourceBook claimed that there were 2 quaternaries of numbers, one which is made by addition, the other by multiplication; and these quaternaries encompass the musical, geometric and arithmetic ratios of which the harmony of the universe so composed. The first quartenary is 1,2,3,4. There are 11 total quartenaries. And the perfect world which results from these quaternaries is geometrically, harmonically and arithmetically arranged"

It is said that the Pythagorean musical system was based on the Tetractys as the rows can be read as the ratios of 4:3, 3:2, 2:1, forming the basic intervals of the Pythagorean scales. Pythagorean scales are based on pure fifths (in a 3:2 relation), and pure fourths (in a 4:3 relation) which form a stable optimally blending intervals. The ratios of 1:1 and 2:1 generate stable purely blending intervals.

Quote:

"The Tetractys [also known as the decad] is an equilateral triangle formed from the sequence of the first ten numbers aligned in four rows. It is both a mathematical idea and a metaphysical symbol that embraces within itself — in seedlike form — the principles of the natural world, the harmony of the cosmos, the ascent to the divine, and the mysteries of the divine realm. So revered was this ancient symbol that it inspired ancient philosophers to swear by the name of the one who brought this gift to humanity — Pythagoras."
— Attributed to Iamblichus

Tetractys - Kabbalist symbol

There are some who believe that the tetractys and its mysteries influenced the early kabbalists. A Hebrew Tetractys in a similar way has the letters of the Tetragrammaton (the four lettered name of God in Hebrew scripture) inscribed on the ten positions of the tetractys, from left to right. It has been argued that the Qabalistic Tree of Life, with its ten spheres of emanation, is in some way connected to the tetractys, but its form is not that of a triangle.

Tetractys - Tarot card reading arrangement

In a Tarot reading, the various positions of the tetractys provide a representation for forecasting future events.
The first row of a single position represents the Premise of the reading, forming a foundation for understanding all the other cards.
The second row of two positions respresents the cosmos and the individual and their relationship
.

- The Light Card to the right represents the influence of the cosmos leading the individual to an action.
- The Dark Card to the left represents the reaction of the cosmos to the actions of the individual.

The third row of three positions represents three kinds of decisions an individual must make.

- The Creator Card is rightmost, representing new decisions and directions that may be made.
- The Sustainer Card is in the middle, representing decisions to keep balance, and things that should not change.
- The Destroyer Card is leftmost, representing old decisions and directions that should not be continued.

The fourth row of four positions represents the four Greek elements.

- The Fire card is rightmost, representing dynamic creative force, ambitions, and personal will.
- The Air card is to the right middle, representing the mind, thoughts, and strategies toward goals.
- The Water card is to the left middle, representing the emotions, feelings, and whims.
- The Earth card is leftmost, representing physical realities of day to day living.

Pentagram

A pentagram is a five-pointed star drawn with five straight strokes. In fact, the word *pentagram* comes from the Greek word πεντάγραμμον (*pentagrammon*), a noun form of πεντάγραμμος (*pentagrammos*) or πεντέγραμμος (*pentegrammos*), a word meaning roughly "five-lined" or "five lines".

The name indicates that a pentagram is not simply a five-pointed star; the symbol *must* be composed of five lines. That is, it must include the interior pentagon.

It is also known as a *pentacle*, *pentalpha* (as it can be formed by five A's) or *pentangle*.

Some sources assert that **pentacle** derives from the French root *pend-* to hang (compare pendant) and means some sort of physical object such as an amulet; this explains why some groups use pentacles that do not have pentagrams inscribed upon them, as attested in the Theosophical Society's *Encyclopedic Theosophical Glossary*[1] But some dictionaries give the etymology as from Italian *pentacolo* or Medieval Latin *pentaculum*, ultimately from Greek *pente*. However, there is Latin usage as early as 1600 showing that a pentacle or pentaculum need not have a five pointed star on it.[2] See also *Man, Myth & Magic* vol. 77. Richard Cavendish, ed. Purnell for BPC Publishing Ltd., London: 1971. p. 2159.

The pentagram has long been associated with the planet Venus and the worship of the goddess Venus, or her equivalent. It is most likely to have originated from the observations of prehistoric astronomers. When viewed from Earth, successive inferior conjunctions of Venus plot a nearly perfect pentagram shape around the Sun every eight years.

Some writers refer to the pentagram as the *endless knot* and sometimes as the *seal of Solomon* (although that is more commonly depicted as a six-pointed star).

Pentagrams were used symbolically in ancient Greece and Babylonia. The Pentagram has magical associations, and many people who practice pagan faiths wear them. Christians once commonly used the pentagram to represent the five wounds of Jesus, but its more modern associations are with Neopagans and Satanists.

Pentagram - Geometry

A regular pentagram is the {5/2} star polygon. It is most easily drawn by drawing a regular pentagon, joining the corners with lines and erasing the original pentagon. You may also extend the sides of the pentagon until they meet, obtaining a bigger pentagram.

The golden ratio, $\varphi = (1+\sqrt{5})/2 = 1.618\ldots$, satisfying

plays an important role in regular pentagons and pentagrams. Each line is divided into several smaller segments, and if you divide the length of the longer segment with the shorter segment of any pair of segments you will get φ.

Also a side of the large pentagon is a blue line, while a diagonal of the small pentagram is the same length as a green line segment.

Like a regular pentagon, and a regular pentagon with a pentagram inside, etc., the regular pentagram has as symmetry group the dihedral group of order 10.

If the lines have thickness, there are two similar regular pentagons. How the "lines", now bars, and their crossings are drawn may vary. For example, on the flag of Morocco (see below), the green pentagram has a black outline which does not "cross" the crossing bars. This is different on the flag of Ethiopia, where the pentagram is plain yellow without outline.

Pentagram - Some relevant trigonometric values

As a result, in an isosceles triangle with one or two angles of 36°, the longer of the two side lengths is φ times that of the shorter of the two, both in the case of the acute as in the case of the obtuse triangle.

complete graph; the complete graph on five vertices can be drawn as a pentagram, heptagram, hexagram and Seal of Solomon, magic and magick, Morning Star, mullet, pentagon, Pythagoras, red star, star (glyph), List of symbols, Nonconvex uniform polyhedra with full icosahedral_symmetry (many show a pattern of pentagrams), Order of the Eastern Star

Pentagram - History

The first uses of the pentagram we know of are found in Mesopotamian writings dating to about 3000 B.C. The Sumerian pentagrams served as pictograms for the word "UB," meaning "corner, angle, nook; a small room, cavity, hole; pitfall," suggesting something very similar to the pentemychos (see below on the Pythagorean use for what pentemychos means). In the Labat (dictionary of Sumerian hieroglyphs/pictograms) it is the number 306, and it is shown as being two points up. In the Babylonian context, the edges of the pentagram were probably orientations: forward, backward, left, right, and "above". These directions also had an astrological meaning, representing the five planets Jupiter, Mercury, Mars and Saturn, and Venus as the "Queen of Heaven" (Ishtar) above.

Pentagram - Pythagorean use

The Pythagoreans called the pentagram ύγιεια *Hygieia* ("health" also the Greek goddess of health, Hygieia), and saw in the pentagram a mathematical perfection (see also above).

The five vertices were also used by the medieval neo-pythagoreans (whom one could argue were not pythagoreans at all) to represent the five Classical elements:

- ύδωρ, *Hydor*, water
- Γαια, *Gaia* earth
- ίδέα, Idea or ίερόν, *Hieron* "a divine thing"
- έιλή, *Heile*, heat (fire)
- άήρ, *Aer*, air

The vertices were labeled in the letters of υ-γ-ι-ει-α though the ordering (clockwise or counter-clockwise) used, and starting vertex, could vary.

Heinrich Cornelius Agrippa, and others, recognized the letters as being the five beginning letters of the words: udor (water, often transliterated as hydor), ge (earth, Agrippa used gaia), idea (idea as in The Platonic Idea), eile (sometimes written as heile, heat, it meant that in the Greek known to Agrippa), and aer (air). However, note that even if one uses the "elemental" scheme that Agrippa used, it's a definite stretch because it uses heat (heile), which is not an element but a quality OF the element fire. Had Agrippa used the Attic Greek, which is the language Pythagoreans back then actually spoke and wrote, he could have used (phonetic) "empreesis" for fire or conflagration.

The ancient, and one could argue real, Pythagorean pentagram was two points up and represented the doctrine of *Pentemychos*. Pentemychos was the title of a work written by Pythagoras' teacher and friend Pherecydes of Syros. Pentemychos means five "recesses" or "chambers" also known as the pentagonas - the five-angle. (This is actually a lost book whose contents are preserved in *Damascius de principiis*, quoted in Kirk and Raven, The Pre-Socratic Philosophers, Cambridge Univ. Press, 1956, page 55). It was also the "place" where the first pre-cosmic offspring had to be put in order for the ordered cosmos to appear. The pentemychos is in Tartaros.

In very early Greek thought, Tartaros (or Chaos, according to Hesiod) was the first existing Darkness from which the cosmos is born. While it was locked away after the emergence and ordering of the cosmos, it still continued to have an influence. In fact, it was known as "the subduer of both gods and men" (Homer), and it was from this that the world got its "psyche" (soul) and its "daimon". The Boundless Darkness held influence through Mychos or Krater. Apart from being the gateway from "there" to "here" it was also a way in the opposite direction, from "here" to "there", as is evident in the many tales about how Greek heroes, philosophers and mystics descended through Krater to Tartaros/Hades (the distinction between the two was very optional back then) in quest for Wisdom. The Underworld as the source of wisdom was the rule.

Tartaros was also later seen as the "chthonic realm" where all the enemies of the cosmic order were locked away, also called the "prison-house" of Zeus. It was said to lay outside of the aither over which Zeus had lordship; what we today would call space, back then called "Zeus' defense-wall," yet it was also beneath the earth. Plato (in Cratylus) said that the aither had a penetrating power that permeates the whole world, and he found it both inside and outside of our bodies. The pentemychos is outside, or in-side, of the aither.

In the play Medea by Euripides, the sorceress Medea calls upon Hecate with the words, "By that dread queen whom I revere before all others and have chosen to share my task, by Hecate who dwells within my inmost chamber, not one of them shall wound my heart and rue it not." Note

that she speaks of the Heart. The inmost chamber is the Mychos. Normally, Hecate and Persephone are portrayed as just being the rulers of the Underworld, however, Hecate is called the Lady of Tartaros, Phulada (Guardian), Propulaia (Before the Gates), Kleidophoros (Key-bearer) and Kleidoukhos (Key-holder, Priestess) - but this Underworld of the Greeks and Pythagoreans is also the "inmost chamber" and the Core of Inner Being.

In the image above of the two points up pentagram from Agrippa's book, the notion of Health that is within and inside a living being, is "bounded between the rings". According to myth, Tartaros was locked away by iron (fashioned by Zeus) and bronze (fashioned by Zeus' brother Poseidon). The iron of Zeus is the aither spoken of earlier ("space") and the bronze is demiurgos, or the waterforce which binds things into matter (Poseidon was often referred to as "he who shakens the earth" and the god Proteus, who carries the seals of Poseidon has ever since Antiquity been seen as a representation of matter.) To sum it up; even if seen as the medieval elements, Living Things are the things traveling the Wheel of Life, and *not* directly connected to or part of the Pythagorean pentagram itself, which is at their core in the center, a Core of Inner Being (the "soul" of "the stuff"). When one had Health, they were considered Zoös and in their hearts there was Eros (the word did not mean "erotic" in the sense we mean it today at all). To not have Health was to be Thanatos - a word that does not exactly mean dead as in "dead and buried," but means dead inside but alive, like a shell.

This is an esoteric connection, but note that the Core is the Greek Goddess Kore (another name for Persephone, alongside Hecate queen of the underworld), whose symbol was the apple. If you cut an apple transversally through its core, it reveals a pentagram. The apple is also still, despite Christian ideas over the years, a symbol of health. (An apple a day keeps the doctor away.)

Quincunx

For Sir Francis Galton's machine for demonstrating the normal distribution named "quincunx", see bean machine. For the Roman coin denomination, see quincunx (coin).

A **quincunx** is the arrangement of five units in the pattern corresponding to the five-spot on dice, playing cards, or dominoes. A quincunx looks like this:

The quincunx pattern originates from Pythagorean mathematical mysticism. This pattern lies at the heart of the Pythagorean tetraktys, a pyramid of ten dots. To the Pythagoreans the number five held particular significance and the quincunx pattern represented this. Sir Thomas Browne moulds his mystical discourse *The Garden of Cyrus* (1658) on the quincunx pattern.

The power of the Pythagorean mysteries is based upon a mystical understanding of the mathematical order of the Universe which could be summed up in visual representation of such numbers as the Tetraktys (10) and the Quincunx (5). - Robert Graves

, *The White Goddess*

- The quincunx is named after the Roman coin of the same name, which was marked with a quincunx of dots.
- A quincunx was the standard tactical formation for elements of a Roman legion.
- A quincunx is a standard pattern for planting an orchard, especially in France.
- Quincunxes are used in modern computer graphics as a sampling pattern for astochastic anti-aliasing.
- In astrology (and less commonly in astronomy), a quincunx is an angle of five-twelfths of a circle, or 150°, between two objects (the Sun, Moon or planets). In astrology this astrological aspect (the term by which specific angles between objects is known) is considered somewhat unfavorable, but less so than the aspects of opposition (180° angle) or square (90° angle).

Appendix

The Marcosian System

Though the Gnostics were derided by the Pythagoreans, the Marcosian system of Gnosticism stands a bit apart. The Marcosian system was a degraded variety of that of Valentinus. It retained the 30 Æons, but called them "Greatnesses" and gave them numerical values. It kept the myth of the fall of Sophia but called it a "Divine Deficiency". But peculiar to this system was the adaptation of the Pythagorean number theory to its cosmogony. The 30 Æons are obtained by adding the numbers of the Ogdoad together: 1+2+3+4+5+7+8 = 30. The 6 is purposely omitted for it is the *episemon* and not a letter of the usual Greek alphabet. The fall of Sophia is clearly shown by the fact that *Lambda* which equals 30, or the complete set of Greatnesses, is really only the eleventh letter of the alphabet, but to make up for this deficiency it sought a consort and so became M (= *Lambda Lambda*). The *episemon*, or 6, is a number full of potency; the name *Iesous* consists of six letters, hence the name of the Saviour. When the Propator, who is the *Monas*, willed the Unspeakable to be spoken, He uttered the Word which has 4 syllables and 30 letters. The plenitude of Greatness is 2 tetrads, a decad and a dodecad (4+4+10+12 = 30); the 2 tetrads are the Unspeakable, Silence, Father and Truth followed by Logos, Life, Man and Church. These form the Ogdoad. The mutes of the Greek alphabet belong to Father and Truth (The Unspeakable, and Silence, of course, do not count); these being mute reveal nothing to man. The semivowels belong to Word and Life, but the vowels to Man and Church, for through Man voice gave power to all. The 7 Greek vowels go through the seven heavens, which thus sing the Great Doxology in harmony. Even numbers are female, odd numbers male, by the union of the first of these, 2 3, was begotten the episemon, or 6, the number of our Salvation.

Apollonius of Tyana

Apollonius of Tyana (13 March 2 – 98?) was a Neo-Pythagorean philosopher and teacher of Greek origin. His teaching influenced scientific thought for centuries after his death.
He is best known through the medium of the writer Philostratus, in whose biography some have seen an attempt to construct a rival to Jesus Christ. Apollonius was a vegetarian, and a disciple of Pythagoras. He is quoted as having said "For I discerned a certain sublimity in the discipline of Pythagoras, and how a certain secret wisdom enabled him to know, not only who he was himself, but also who he had been; and I saw that he approached the altars in purity, and suffered not his belly to be polluted by partaking of the flesh of animals; and that he kept his body pure of all garments woven of dead animal refuse; and that he was the first of mankind to restrain his tongue, inventing a discipline of silence described in the proverbial phrase, "An ox sits upon it." I also saw that his philosophical system was in other respects oracular and true. So I ran to embrace his teachings..."
He was born in the city of Tyana, in the Roman Empire province of Cappadocia in Asia Minor, a few years before the Christian era. He was educated in the nearby city of Tarsus and in the temple of Asclepius at Aegae, where he devoted himself to the doctrines of Pythagoras and adopted the ascetic habit of life in its fullest sense.
After keeping a vow of silence for five years, he trekked from Greece through Asia and visited Nineveh, Babylon and India, imbibing the oriental mysticism of magi, Brahmans and gymnosophists. During this journey, and subsequent return, he attracted a scribe and disciple, Damis, who recorded events in the life of the philosopher. These notes described a number of events in the life of Apollonius, including events relating to a succession of emperors, as he lived 100 years. Eventually the notes of Damis came into the possession of the empress Julia Domna wife of Severus, who commissioned Philostratus to use the notes to assemble a biography of the sage.
The narrative of his travels given by his disciple Damis and reproduced by Philostratus is so full of the miraculous that many have regarded him as an imaginary character. On his return to Europe he was saluted as a magician, and received the greatest reverence from priests and people generally. He himself claimed only the power of foreseeing the future; yet in Rome it was said that he raised from death the daughter of a senator. In the halo of his mysterious power he passed through Greece, Italy and Spain. It was said that he was accused of treason both by Nero and by Domitian, but escaped by miraculous means. Finally he set up a school at Ephesus, where he died, apparently at the age of a hundred years. Philostratus keeps up the mystery of his hero's life by saying, "Concerning the manner of his death, *if he did die*, the accounts are various."
The work of Philostratus, is generally regarded as a religious work of fiction. It contains a number of obviously fictitious stories, through which, however, it is not impossible to discern the general character of the man. In the 3rd century, Hierocles endeavoured to prove that the doctrines and the life of Apollonius were more valuable than those of Christ, and, in modern times, Voltaire and Charles Blount (1654-1693), the English freethinker, have adopted a similar standpoint. Apart from this extravagant eulogy, it is absurd to regard Apollonius merely as a vulgar charlatan and miracle-monger. If we cut away the mass of mere fiction which Philostratus accumulated, we have left a highly imaginative, earnest reformer who laboured to infuse into the flaccid dialectic of paganism a saner spirit of practical morality.
He wrote many books and treatises on a wide variety of subjects during his life, including science, medicine, and philosophy.

His scientific theories were ultimately applied to the earth-centered Ptolemaic idea that the sun revolved around the earth. A few decades after his death, the Emperor Hadrian collected his works and ensured their publication throughout his realm.

Some scholars, both ancient and contemporary, believe that Apollonius was actually the Christian Apostle Paul, as many of his teachings coincide with those of Paul, and Apollonius is said to have done many of the same things Paul did. Others suggest Apollonius was the Jesus Christ of the Christian scriptures. Others, such as the 2nd-century wit Lucian, ascribed the false magic of "the notorious Apollonius", transmitted through a pupil, to the bag of impostures played by *Alexander the false prophet*, a contemporary travelling mage posing as a priest of Asclepius, ca 150 – 170.

Apollonius' fame was still evident in 272, when the Emperor Aurelian besieged Tyana, which had rebelled against Roman rule. In a dream or vision, Aurelian claimed to have seen Apollonius speak to him, beseeching him to spare the city of his birth. In part, Aurelian said Apollonius told him "Aurelian, if you desire to rule, abstain from the blood of the innocent! Aurelian, if you will conquer, be merciful!" Aurelian, who admired Apollonius, spared Tyana.

Medieval Islamic alchemist Jabir ibn Hayyan's *Book of Stones* is a lengthy analysis of alchemical works attributed to Apollonius (called "Balinas") (see e.g. Haq, which provides an English translation of much of the *Book of Stones*).

Archytas

Archytas (428 BC - 347 BC), was a Greek philosopher, mathematician, astronomer, statesman, strategist and commander-in-chief.

Archytas was born in Tarentum, Magna Graecia (now Italy) and was the son of Mnesagoras or Histiaeus. He was taught for a while by Philolaus and he was a teacher of mathematics to Eudoxus of Cnidus. He was scientist of the Pythagorean school, famous as the intimate friend of Plato. His and Eudoxus' student was Menaechmus.

Sometimes he is believed to be the founder of mathematical mechanics.

He is also reputed to have designed and built the first artificial, self-propelled flying device, a bird-shaped model propelled by a jet of what was probably steam, said to have actually flown some 200 yards. This machine, which its inventor called The Pigeon, may have been suspended on a wire or pivot for its "flight".

According to Eutocius Archytas solved the problem of duplicating the cube in his manner with a geometric construction

. Hippocrates of Chios before reduced this problem to finding mean proportionals. Archytas' theory of proportions is treated in the book VIII. of Euclid's *Elements*.

The Archytas curve, which he used in his solution of the doubling the cube problem, is named after him.

Archytas was drowned in the Adriatic Sea; his body lay unburied on the shore till a sailor humanely cast a handful of sand on it, otherwise he would have had to wander on this side the Styx for a hundred years, such the virtue of a little dust, *munera pulveris*, as Horace calls it.

Biography of ARCHYTAS [375 B.C.]

by D1OGENES LAERTES [180 A.D.]

(From Chaignet)

Archytas of Tarentum, son of Mnesagoras, or of Hestius, according to Aristoxenus, also was a Pythagorean. It was he who, by a letter, saved Plato from death threatened by Dionysius. He possessed all the virtues, so that, being the admiration of the crowd, he was seven times named general, in spite of the law which forbad re-election after one year. Plato wrote him two letters, in response to this one of Archytas:

"Greetings. It is fortunate for you that you have recovered from your illness; for I have heard of it not only from you, but also from [Lamiscus]. I have busied myself about those notes, and took

a trip into Lucania, where I met descendants of Ocellus. I have in my possession the treatises *on Law and Royalty*; *on Holiness*, and *on the Origin of All Things*; and I am sending them to you. The others could not be discovered. Should they be found, they will be sent to you."

Plato answered:

"Greetings. I am delighted to have received the works which you have sent me, and I acknowledge a great admiration for him who wrote them. He seems to be worthy of his ancient and glorious ancestors, who are said to be [Tyreans] and among the number of those Trojans who emigrated under the leadership of Laomedon, [all] worthy people, as the legend proves. Those works of mine about which you wrote me are not in a sufficient state of perfection, but I send them such as they are. Both of us are in perfect agreement on the subject of protecting them. No use to renew the request. May your health improve!"

Such are these two letters. There were four Archytases. The first, of whom we have just spoken. The second, from Mytilene, was a musician; the third wrote about agriculture; fourth is an author of epigrams. Some mention a fifth; an architect, who left a treatise on mechanics, beginning as follows: This book contains what I have been taught by the Carthaginian Teucer.

The musician is said to have made this joke. Being reproached for not advertising himself more, he said: It is my instrument, which speaks for me. Aristoxenus claims that the philosopher Archytas was never vanquished when he commanded. Once, overcome by envy, he had been obliged to resign his command; and his fellow-citizens were immediately conquered. He was the first who methodically applied the principles of mathematics to mechanics; who imparted an organic motion to a geometric figure, by the section of the semi-cylinder seeking two means that would be proportional, to double the cube. He also first, by geometry discovered the properties the cube, as Plato records in the Republic.

SECTION I

METAPHYSICAL FRAGMENTS

(Stob.Ec.Phys. 1:-?13)

1. There are necessarily two principles of beings; the one containing the series of beings organized, and finished, the other, of unordered and unfinished beings. That one which is susceptible of being expressed, by speech, and which can be explained, both embraces beings, and determines and organises the non-being.

For every time that it approaches the things of becoming, it orders them, and measures them, and makes them participate in the essence and form of the universal. On the contrary, the series of beings which escape speech and reason, injures ordered things and destroys those which aspire to essence and becoming; whenever it approaches them, it assimilates them to its own nature.

But since there are two principles of things of an opposite character, the one the principle of good, and the other the principle of evil, there are therefore also two reasons, the one of beneficent nature, the other of maleficent nature.

That is why the things that owe their existence to art, and also those which owe it to nature, must above all participate in these two principles; form and substance.

The form is the cause of essence; substance is the substrate which [it] receives the form. Neither can substance alone participate in form, by itself; nor can form by itself apply itself to substance; there must therefore exist another cause which moves the substance of things; and forms them. This cause is primary, as regards substance, and the most excellent of all. Its most suitable name is God.

There are therefore three principles: God, the substance of things, and form. God is the artist, the mover; the substance is the matter, the moved ; the essence is what you might call the art, and that to which the substance is brought by the mover. But since the mover contains forces which are self-contrary, those of simple bodies, and as the contraries are in need of a principle harmonizing and unifying them, it must necessarily receive its efficacious virtues and proportions from the numbers, and all that is manifested in numbers and geometric forms; virtues and proportions capable of binding and uniting into form the contraries that exist in the substance of things. For, by itself, substance is formless; only after having been moved towards form does it become form, and receives the rational relations of order. Likewise, if movement exists, besides the thing moved, there must exist a prime mover; there must therefore be three principles; the substance of things, the form, and the principle that moves itself, and which by its power is the first; not only must this principle be an intelligence, it must be above intelligence, and we call it God.

Evidently the relation of equality applies to the being which can be defined to language, and reason. The relation of inequality applies to the irrational being; and cannot be fixed by language; it is substance; that is why all begetting and destruction take place in substance, and do not occur without it.

2. In short, the philosophers began only by so to speak contrary principles; but above these elements they knew another superior one, as is testified to by Philolaus, who says that God has produced, and realized the finite and infinite, and shown that at the limit is attached the whole series which has a greater affinity with the One, and to Infinity, the one that is below. Thus, above these two principles they have posited a unifying cause, superior to everything which, according to Archenetus, is the cause before the cause, and, according to Philolaus, the universal principle.

3a. Which unity are you referring to? Of supreme unity, or of the infinitely small unity that you can find in the parts? The Pythagoreans distinguished between the Unity and the *Monad*, as says Archytas ; Unity and the *Monad* have a natural affinity but yet they differ.

3b. Archytas and Philolaus indiscriminately call the unity a *monad*, and *monad* a unity. The majority however add to the same *monad*, the distinction of first *monad*, for there is a *monad* which is not the first, and which is posterior to the *monad* in itself, and to unity.

3c. Pythagoras said that the human soul was a tetragon with right angles. Archytas, on the contrary, instead of defining the soul by the tetragon, did so by a circle, because the soul is a self-mover, and consequently, the prime mover; but this [is] a circle or a sphere.

3d. Plato and Archytas and the other Pythagoreans claim that there are three parts in the soul; reason, courage and desire.

4. The beginning of the knowledge of beings is in the things that produce themselves. Of these some are intelligible, and others sensible; the former are immovable, the latter are moved. The criterion of intelligible things is the World; that of sensible things is sensation.

Of the things that do not manifest in things themselves, some are science, the others, opinion; science is immovable; opinion is movable. We must, besides, admit these three things; the subject that judges, the object that is judged, and the rule by which that object is judged. What judges, is the mind, or sensation; that is judged, is the *logos* or rational essence; the rule of judgment is the act itself which occurs in the being; whether intelligible or sensible. The mind is the judge of essence, whether it tends towards an intelligible being, or a sensible one. When reason seeks intelligible things, it tends towards an intelligible element; when it seeks things of sense, it tends towards their element. Hence come, those false graphic representation in figures and numbers seen in geometry, those researches in causes and probable ends, whose object are beings subject to becoming, and moral acts, ... physiology or politics. It is while tending towards the intelligible element that reason recognizes that harmony is in the double relation; but sensation alone attests that this double relation is concordant. In mechanics, the object of science is figures, numbers, proportions; -- namely rational proportions; the effects are perceived by sensation; for you can neither study nor know them outside of the matter or movement. In short it s impossible to know the reason of an individual thing, unless you have preliminarily by the mind grasped the essence of the individual thing. The knowledge of the existence, and of quality, belongs to reason and sensations; to reason, whenever we effect a thing's demonstration by a *syllogism* whose conclusion is inevitable; to sensation, when the latter is the criterion of a thing's essence.

5. Sensation occurs in the body, reason in the soul. The former is the principle of sensible things, the latter, of intelligible ones. Popular measures are number, length, the foot, weight and equilibrium; the scales; while the rule and the measure of straightness in both vertical and longitudinal directions is the right angle.

Thus sensation is the principle and measure of the bodies; reason is the principle and measure of intelligible things. The former is the principle of beings that are intelligible and naturally primary; the latter, of sense-objects, and naturally secondary. Reason is the principle of our soul; sensation, the principle of our body. The mind is the judge of the noblest things; sensation, of the most useful. Sensation was created in view of our bodies, and to serve them; reason in view of the soul, and to initiate wisdom therein. Reason is the principle of science; sensation, of opinion.

The latter derives its activity from sensible things; the former from the intelligible. Sensible objects participate in movement and change; intelligible objects participate in immutability and eternity. There is no analogy between sensation and reason; for sensation's object is the sensible, which moves, changes, and never remains self-identical; therefore as you can see it, it improves or deteriorate. Reason's object is the intelligible; whose essence is immobility, wherefore in the intelligible we cannot conceive of either more nor less, better or worse; and just as reason sees the primary being, and the (cosmic) model, so sensation sees the image, and the copied. Reason sees man in himself; sensation sees in them the circle of the sun, and the forms of artificial objects. Reason is perfectly simple and indivisible, as unity, and the point; it is the same with intelligible beings.

The idea is neither the limit nor the frontier of the body; it is only the figure of being, that by which the being exists, while sensation has parts, and is divisible.

Some beings are perceived by sensation, others by opinion, others by science, and others by reason.

The bodies that offer resistance are sensible; opinion knows those that participate in the ideas, and are its images, so to speak. Thus some particular man participates in the idea of man, and this triangle, in the triangle-idea. The object of science are the necessary accidents of ideas; thus the object of geometry is the properties of the figures; reason knows the ideas themselves, and the principles of the sciences and of their objects; for example, the circle, the triangle, ,and the pure sphere in itself. Likewise, in us, in our souls, there are four kinds of knowledge, pure thought, science, opinion and sensation; two are principles of knowledge (thought and sensation); two are its purpose, science and opinion. It is always the similar which is capable of knowing the similar; reason knows the intelligible things; science, the knowable things; opinion, conjecturable things; sensation, sensible things.

That is why thought must rise from things that are sensible, to the conjecturable ones and from these to the knowable, and on to the intelligible and he who wishes to know the truth about these objects, must in a harmonious grouping combine all these means and objects of knowledge. This being established, you might represent them under the image of a line divided into two equal parts, each of which would be similarly divided; if we separate the sensible, dividing it into two parts, in the same proportion, the one will be clearer, the other obscurer. One of the sections of the of the sensible contains images of things, such as you see reflected in water, or mirrors; the second represents the plants and animals of which the former are images. Similarly dividing the intelligible, the different kinds of sciences will represent the images; for the students of geometry begin by establishing by hypothesis, the odd and the even, figures, three kinds of angles, and from these hypotheses deduce their science; as to the things themselves, they leave them aside, as if they knew them, though they not cannot account for them to themselves or to others; they employ sensible things as images, but these things are neither the object nor the end proposed in their researches and reasonings; which pursue only things in themselves, such as the diameter or square. The second section is that of the intelligible; object of dialectics. It really makes no hypotheses, positing principles whence it rises to arrive to the unconditioned, to the universal principle; then, by an inverse movement, grasping that principle, it descends to the end of the reasoning, without employing any sensible object, exclusively using pure ideas. By these four

divisions, you can also analyse the soul-states, and give the highest the name of thought, reasoning to the second, faith to the third, and imagination to the fourth.

6. Archytas, at the beginning of his book *on Wisdom* gives this advice; in all human things wisdom is as superior as sight is to all the other senses of the body, as mind is superior to soul, as the sun is superior to the stars. Of all the senses, sight is the one that extends furthest in its sphere of action, and gives us the most ideas. Mind, being supreme, accomplishes its legitimate operation by reason and reasoning; it is like sight, and the power of the noblest objects; the sun is the eye and soul of natural things, for it is through it that they are all seen, begotten, and thought; through it the beings produced by root or seed, are fed, developed, and endowed with sensation. Of all beings, man is the wisest; by far; for he is able to contemplate beings, and to acquire knowledge and understanding of all. That is why divinity has engraved in him, and has revealed to him the system of speech, which extends to everything, a system in which are classified all the beings, kinds of being, and the meanings of nouns and verbs. For the specialised seats of the voice are the pharynx, the mouth and the nose. As man is naturally organized to produce sounds, through which nouns and verbs are expressed and formed, likewise he is naturally destined to contemplate the notions contained in the visible objects; such, in my view, is the purpose for which man has been created, and was born; and for which he received from God his organs and faculties.

Man is born and is created to know the essence of universal nature; and precisely the function of wisdom is to possess and contemplate the intelligence manifested in the beings.

The object of wisdom is no particular being, but all the beings, absolutely; and it should not begin to seek the principles of an individual being, but the principles common to all. The object of wisdom is all the beings, as the object of sight is all visible things. The function of wisdom is to see all the beings in their totality, and to know their universal attributes; and that is how wisdom discovers the principles of all beings.

He who is capable of analysing all the species, and to trace and group them, by an inverse operation, into one single principle, he seems to me the wisest, and the closest to the truth; he seems to have found that sublime observatory from the peak of which he may observe God, and all the things that belong to the series and order of divine things; being master of this royal road, his mind will be able to rush forwards, and arrive at the end of the career, uniting principles to the purposes of things, and knowing that God is the principle, the middle and the end of all things made according to the rules of justice and right reason.

SECTION II

PHYSICAL AND MATHEMATICAL FRAGMENTS

7. As Eudemus reports, Archytas used to ask this question: If I was situated at the extreme and immovable limit of the world, could I, or not, extend a wand outside of it? To say I could not, is absurd; but if I can, there must be something, outside of the world, be it body or space; and in whatever manner we reason, by the same reasoning we will ever return to this limit. I will still place myself there, and ask, is there anything else on which I may place my wand. Therefore, the

infinite exists; if it is a body, our proposition is demonstrated; if it its space, place is that in which a body could be; and if it exists potentially, we will have to place it among, classify it among the eternal things, and the infinite will then be a body and a place.

8. The essence of place is that all other things are in it, while itself is not in anything. For if it was in a place, there would be a place in a place, and that would continue to infinity. All other beings must therefore be in place, and place in nothing. Its relation to things is the same as limit to limited things; for the place of the entire world is the limit of all things.

9a. Some say that time is the sphere of the world; such was the sentiment of the Pythagoreans, according to those who had no doubt heard Archytas give this general definition of time: "Time is the interval of the nature of all."

9b. The divine Iamblichus, in the first book of his commentaries on the Categories, said that Archytas thus defined time: "It is the number of movement, or in general the interval of the nature of all."

9c. We must combine these two definitions, and recognise time as both continuous and discrete, though it is properly continuous. Iamblichus claims that Archytas taught the distinction of time physical, and time psychic; so at least Iamblichus interpreted Archytas; but we must recognise that there, and often elsewhere, he adds to his commentaries to explain matters.

10. The general proper essence of "When-ness" and time is to be indivisible and insubstantial. For, being indivisible, the present time has passed, while expressing it, and thinking of it; naught remains of it, becoming continuously the same, it never subsists numerically, but only specifically. In fact, the actually present time and the future are not identical with former time. For the one has past, and is no more; the other passes while being produced and thought. Thus the present is never but a bond; it perpetually becomes, changes, and perishes; but nevertheless it remains identical in its own kind.

In fact, every present is without parts, and indivisible; it is the term of past time, the beginning to come; just as in a broken line, the point where the break occurs becomes the beginning of a line, and the end of the other. Time is continuous, and not discrete as are number, speech and harmony. In speech, the syllables are parts, and distinct parts; in the harmony, they are the sounds; in number; the unities. The line, place and space are continuous; if they are divided, their parts form common sections. For the line divides into points, the surface into lines, the solid into surfaces. Therefore time is continuous. In fact, there was no nature, when time was not; and there was no movement, when the present was not. But the present has always been, it will always be, and will never fail; it changes perpetually, and becomes an other according to the number, but remains the same according to kind. The line differs from the other continua, in that if you divide the line, place, and space, its parts will subsist; but in time, the past has perished, and the future will. That is why either time does absolutely not exist, or it hardly exists, and has but an insensible existence. For of its parts one, the past, is no more; the future is not yet, how then could the present, without parts and indivisible, possess true reality?

11. Plato says that the movement is the great and the small, the non-being. the unequal and all that reduces to these; like Archytas we had better say that it is a cause.

12.Why do all natural bodies take the spherical form? Is it, as said Archytas, because the natural movement is the proportion of equality? For everything moves in proportion; and this proportion of equality is the only one which, when it occurs, produces circles and spheres because it returns on itself.

13. He who knows must have learned from another, or have found his knowledge by himself. The science that you learn from another, is as you might say, exterior; what you find by yourself belongs to ourselves individually. To find without seeking is something difficult and rare; to find what one is seeking is commodious and easy; to ignore, and seek what you ignore, is impossible.

14. The Pythagorean opinion about sciences to me seems correct, and they seen to show an exact judgment about each of then. Having known how to form a just idea of the nature of all, they should have likewise seen the essential nature of the parts. They have left us certain evident theories about arithmetic, geometry, spherics; also about music; for all these sciences seem to be kindred, in fact, the first two kinds of being are indistinguishable.

15a. First they have seen that it was not possible that there should be any noise, unless there was a shock of one body against another; they said. There is a shock when moving bodies meet and strike each other. The bodies moved in the air in an opposite direction and those that are moved without equal swiftness, -- in the same direction, -- the first, when overtaken, make a noise, because struck. Many of these noises are not susceptible of being perceived by our organs; some because of the slightness of the shock, the others because of their too great distance from us, some even because of the very excess of their intensity; for noises too great do not enter into our ears, as one cannot introduce anything into jars with too narrow an opening when one pours in too much at a time.

Of the sounds that fall within the range of our senses, same, -- those that come quickly from the bodies struck, seem shrill; those that arrive slowly and feebly, seem of low pitch. In fact, when one agitates some object slowly and feebly, the shock produces a low pitch; if the waving is [done] quickly and with energy, the sound is shrill. This not the only proof of the fact; which we can prove when we speak or sing; when we wish to speak loud and high, we use a great force of breath. So also something thrown; if you throw them hard, they go far; if you throw then without energy, they fall near, for they air yields more to bodies moved with much force, than to those thrown with little. This phenomenon is also reproduced in the sound of the voice; for the sounds produced by an energetic breath are shrill while those produced by a feeble breath are weak and low pitch. This same observation can be seen in the force of a signal given from any place; if you pronounce it loud, it can be heard far; if you pronounce the same signal low, we do not hear it, even from near. So also in flutes, the breath emitted by the mouth and which presents itself to the holes nearest the mouthpiece, produces a shriller sound, because the impulsive force is greater, further, they are of lower pitch. It is therefore evident that the swiftness of the movement produces shrillness and slowness, lower pitch. The same thing in seen in the magic tops which are spun in the mysteries; those that move slowly produce a low pitch, while those that move quickly with force [give] a shrill noise. Let us yet adduce the reed: if you close the lower

opening, and blow into it, it will produce a certain sound; and if you stop it in the centre, or in the front, the sound will be shrill. For the same breath traversing a long space weakens, while traversing a shorter, it remains of the same power. After having developed this opinion that the movement of the voice is measured by the intervals, he resumes his discussion, saying, that the shrill sounds are the result of a swifter movement, the lower sounds, of a slower movement, this is a fact which numerous experiments demonstrate clearly.

15b. Eudoxus and Archytas believed that the reasons of the agreement of the sounds was in the numbers; they agree in thinking that these reasons consist in the movements, the shrill movement being quick, because the agitation of the air is continuous, and the vibration more rapid; the low pitch movement being slow, because it is calmer. 16. Explaining himself about the means, Archytas writes: In music there are three means; the first is the arithmetical mean, the second is the geometrical, the third is the subcontrary mean, which is called harmonic. The mean is arithmetical, when the three terms are in a relation of analogical excess, that is to say, when the difference between the first and second is the same as between second and third; in this proportion, the relation of the greater terms is smaller, and the relation of the smaller is greater. The geometric mean exists when the first term is to the second, as the second is to the third; here the relation of the greater is identical with the relation of the smaller. The subcontrary mean, which we call harmonic, exists when the first term exceeds the second by a fraction of itself, identically with the fraction by which the second exceeds the third; in this proportion, the relation of the greatest terms is greater, and that of the smaller, smaller.

SECTION III

ETHICAL FRAGMENTS

17. We must first know that the good man is not thereby necessarily happy, but that the happy man is necessarily good; for the happy man is he who deserves praise and congratulations; the good man deserves only praise. We praise a man because of his virtue, we congratulate him because of his success. The good man is such because of the goods that proceed from virtue; the happy man is such because of the goods that come from fortune. From the good man you cannot take his virtue; sometimes the happy man loses his good fortune. The power of virtue depends on nobody; that of happiness, on the contrary, is dependent. Long diseases, the loss of our senses cause to fade the flower of our happiness [and luck].

2. God differs from the good man in that God, not only possesses a perfect virtue, purified from all mortal affection, but enjoys a virtue whose power is indefectible, independent, as suits the majesty and magnificence of his works. Man, on the contrary, not only possesses an inferior virtue, because of the mortal constitution of his nature, but even sometimes by the very abundance of his goods, now by the force of habit, by the vice of nature, or from other causes, he is incapable of attaining the perfection of the good.

3. The good man, in my opinion, is he who knows how to act properly in serious circumstances and occasions; he will therefore know how to support good and bad fortune; in a brilliant and glorious condition, he will show himself worthy if it, and if fortune happens to change, he will know how to accept properly his actual fate. In short, the good man is he who, in every occasion,

and according to the circumstances, well plays his part, and knows how to fit to it not only himself, but also those who have confidence on him, and are associated with his fortunes.

4. Since amidst the goods, some are desirable for themselves, and not for anything else, and others are desirable for something else, and not for themselves, there must necessarily exist a third kind of goods, which are desirable both for themselves and for other things. Which are the goods naturally desirable for themselves, and not for anything else? Evidently, it is happiness; for it is the end on account of which we seek everything else, while we seek it only for itself, and not in view of anything else.

Secondly, which are the goods chosen for something else, and not for themselves? Evidently those that are useful, and which are the means of procuring the real goods, which thus become the causes of the goods desirable for themselves; for instance, the bodily fatigues, the exercises, the tests which procure health; reading, meditation, the studies which procure virtues, and the quality of honesty. Last, which are those goods which are both desirable for themselves, and for something else? The virtues, and the habitual possession of virtues, the resolutions of the soul, the actions, and in short anything pertaining to the possession of the beautiful. Thus what is to be desired for itself, and not for anything itself, this is the only good. Now what we seek both for itself and for something else is divided into three classes; the one whose object is the soul, the body, and external goods. The first contains the virtues of the soul, the second the advantages of the body; the third, friends, glory, honor and wealth. Likewise with the goods that are desirable only for something else; one part of them procures goods for the soul, the other which regards the body, procures goods for it; the external goods furnish wealth, glory, honor and friendship.

We can prove that it is characteristic of virtue to be desirable for itself, as follows: In fact, if the naturally inferior goods, I mean those of the body, are by us sought for themselves, and if the soul is better than the body, it is evident that we like the goods of the soul for themselves, and not for the results that they might produce.

5. In human life there are three circumstances: prosperity, adversity, and intermediary comfort. Since the good man who possesses virtue and practises it, practises it in these three circumstances, either in adversity, or prosperity, or comfort, since besides in adversity he is unhappy, in prosperity he is happy and in comfort he is not happy; -- it is evident that happiness is nothing else than the use of virtue in prosperity. I speak here of human happiness. Man is not only a soul, he is also a body; the living being is a composite of both; and man also; for if the body is an instrument of the soul, it is as much a part of the man, as the soul. That is why, among the goods, some belong to the man and others belong to his component parts. The good of man is happiness; amidst its integral parts, the soul's goods are prudence, courage, justice temperance; the body's are beauty, health, good disposition of its members and the perfect condition of its senses. The wealth, glory, honor, nobility, naturally superfluous advantages of man, and naturally subordinate to the superior goods.

The inferior goods serve as satellites to the superior goods; friendship, glory, wealth are the satellites of the body and soul; health, strength and sense-perfection are satellites of the soul; prudence, courage, justice, temperance are the satellites of the reason of the soul; reason is the satellite of God; his is omnipotent, the supreme master. It is for these goods the other must exist;

toe the army obeys the general, [as] sailors to the pilot, the world to God, the [soul] to reason, the happy life to prudence. For prudence is nothing that the science of the happy [---e], or the science of the goods which belong to [--an] nature.

6. To God belong happiness and the happy life;[--] cannot possess but a grouping of science, virtue and prosperity forming a single body. Call wisdom the science for the Gods and [genius]; prudence, the science of human things, the science of life; for science should be the name of virtues which rest on reasons and demonstrations, and moral virtue, the excellent habit of the irrational part of the soul, which makes you move; the name of certain qualities corresponding to our habits, namely the names of liberals, of just men, and of temperate people; and I call prosperity this affluence of goods which we re[quire] without reason, without reason being their cause. Then since virtue and science depend on us, and prosperity does not depend thereon, since happiness consists in the contemplation and practice of good things, and since contemplation and action when they meet obstacles, lend us a necessary support when they go by an easy road, they bring us distraction and happiness; since after all it is prosperity that gives us these benefits, it is evident that happiness is nothing else than the use of virtue in prosperity.

7. Man's relations with prosperity resemble a healthy and vigorous human body; he also can stand heat and cold, raise a great burden, and easily bear many other miseries.

8. Since happiness is the use of virtue in prosperity, let us speak of virtue and prosperity, the latter first. Some goods, such as virtue, are not subject to excess; for excess is impossible in virtue, for one can never be too decent a man; indeed, virtue's measure is duty, and is the habit of duty in practical life. Prosperity is subject to excess and lack, which excesses produce certain evils, disturbing man from his usual mood, so as to oppose him to virtue; this is not only the case with prosperity, but other more numerous causes also produce this effect. You need not be surprised at seeing in the hall certain impudent artists, who neglect true art, misleading the ignorant by a false picture; but do you suppose that this race does not exist as regards virtue? On the contrary, the greater and more beautiful virtue is, the more do people feign to adorn themselves with it. There are indeed many things which dishonor the appearance of virtue; first are the deceivers who simulate it, others are the natural passions which accompany it, and sometimes twist the dispositions of the soul into a contrary direction; others are the bad habits which the body has rooted in us, or have been ingrained in us by youth, age, prosperity, adversity, or a thousand other circumstances. Wherefore we must not at all be surprised at entirely wrong judgments, because the true nature of our soul has has been falsified within us. Just as we see an artist who is excellent make errors in works we are examining; or the general, the pilot or the painter and the like may make errors without our detracting from their talent, so we must not call unworthy him who has had a moment of weakness, nor among the worthy a man who has done no more than a single action; but in respect to the evil, we must consider chance, and for the good, of error, and to make an equitable and just judgment, and not regard a single circumstance, or a single period of time, but the whole life.

Just as the body suffers from both excess and lack, but as nevertheless the excess and so-called superfluities naturally produce the greatest diseases, so the soul suffers of both prosperity and adversity when they arrive at wrong times, and yet the greatest evils come from so-called absolute prosperity that is absolute because like wine it intoxicates the reason of the worthy.

9. That is why it is not adversity but prosperity which is the hardest to stand properly. All men, when they are in adversity, at least greater part of them, seem moderate and modest; and in good fortune, ambitious, vain and proud. For adversity is apt to moderate the soul, and concentrate it; while on the contrary prosperity excites it and puffs it up; that is why the wretches are docile to advice, and prudent in conduct, while the happy are bold and venturesome.

10. There is therefore a measure and limit of prosperity; the one that the worthy man should desire to have as auxiliary in the accomplishment or his actions; just as there is a measure in the size of the ship, and in the length of the tiller; which permits the experienced pilot to traverse an immense extent of sea, and to carry through a great voyage.

The result of excess of prosperity, even among worthy people, is that the soul loses leadership, to prosperity; just as too bright, a light dazzles the eyes, so too great a prosperity dazzles the reason of the soul. Enough about prosperity.

18. I insist that virtue is sufficient to preclude unhappiness, that badness precludes happiness, if we know how properly to judge of the genuine condition of the soul in these two conditions; for the evil is necessarily always unhappy, whether in abundance, -- which he it does not know how properly to judge use, -- or in poverty; just as a blind man is always, whether he is in brilliant light or in darkness. But the worthy man is not always happy; for happiness does not consist in the possession of virtue, but its use; just as a man who sees does not see all the time; he will not see without light.

Life is as it were divided into two roads; the rougher one, followed by patient Ulysses, and the more agreeable one followed by Nestor; I mean that virtue desires the one, but can also follow the other. But nature cries aloud that happiness is life desirable in itself, whose state is assured, because one can realize one's purposes in it, so that if life is traversed by things one has not desired, one is not happy, without however being absolutely unhappy. Therefore be not so bold as to insist that the worthy man is exempt from, sickness, and suffering; dare not to say that he does not know pain; for if the body is allowed some causes of pain, the soul should also be allowed some. The griefs of the insane lack reason and measure while those of the wise are contained within the measure which reason gives to everything; but this so advertised insensibility enervates the character of generosity of virtue, when it stands trials, great sorrows, when it is exposed to death, suffering and poverty; for it is easy to support small sorrows. You must therefore practice the "metriopathy," or sorrow-standardization; so as to avoid the insensibility just as much as the over-sensibility to pain, and not in words to boast about our strength above the measure of our human nature.

19. We might define philosophy as the desire of knowing and understanding things in themselves, joined with practical virtue, inspired and realized by the love of science.

The beginning of philosophy is the science of nature; the middle, practical life; and the end, science itself. It is fortunate to have been well born, to have received a good education, to have been accustomed to obey a just rule and to have habits conformable to nature. One must also have been exercised in virtue, and have been educated by wise parents, governors and masters. It is fine to impose the role of duty on one's self, to have no need of constraint, to be docile to those

who give us good advice about life, and science. For a fortunate disposition of nature, and a good education are often more powerful than lessons to bring us to the good; its only lack would be the efficacious light of reason, which science gives us. Two rival directions of life contend for mastery; practical and philosophical life. By far the most perfect life unites them both, and in each different path adapts itself to circumstances. We are born for rational activity; which we call practical. Practical reason leads us to politics; the theoretical reason, to the contemplation of the universality of things. Mind itself, which is universal, embraces the two powers necessary to happiness, which we define as the activity of virtue in prosperity; it is not exclusively either a practical life which would exclude science, nor a speculative life which would exclude the practical. Perfect reason inclines towards these two omnipotent principles, for which man is born; the principle of society and science; for if these opposite principles seem mutually to interfere in their development, the political principles turning us away from politics, and the speculative principles turning us from speculation, to persuade us to live at rest, nevertheless nature, uniting the ends of these two movements, shows them fused; for virtues are not contradictory and antipathetic mutually; than the harmony of virtues no harmony is more consonant. If, from his youth, man has subjected himself to the principles of virtues, and to the divine law of the world harmony, he will lead an easy life; and if, by his own inclination, he inclines towards evil, and has the luck of meeting better guides, he will, by rectifying his course, arrive at happiness, like passengers favored by chance, finishing a fortunate sea-passage, thanks to the pilot; and the fortunate passage of life is happiness. But if by himself he cannot know his real interests, if he does not have the luck of meeting prudent directors, what benefit would it be if he did have immense treasures ? For the fool, even if he had for himself all the other elements of luck, is eternal unhappy.

And since, in everything, you must first consider the end, -- for that is what is done by the pilots ever meditating over the harbor wither they are to land the ship, and the drivers who keep their eye on the goal of their trip, the archers and slingers who consider their objective, for it is the objective towards which all their efforts must tend, -- virtue must necessarily undertake an objective, which should become the art of living; and that is the name I give it in both directions it can take. For practical life, this objective is improvement; for the philosophical life, the perfect good; which, in their human affairs the sages call happiness. Those who are in misery are not capable of judging of happiness according to exact ideas; and those who do not see it, clearly, would not know how to choose it. Those who consider that pleasure is the sovereign good are punished therefore by foolishness, those who above all seek the absence of pain, also receive their punishment, and, to resume all, to define life-happiness as the enjoyment of the body, or in an unreflective state of soul is to expose himself to all the whirlwinds of the tempest. Those who suppress moral beauty by avoiding all discussion, all reflection about the matter, and seeking pleasure, absence of pain, simple and primitive physical enjoyments, the irreflective inclinations of body and soul, are not more fortunate; for they commit a double fault, by reducing the good of the soul and its superior functions to the level of that of the body, and in raising the good of the body to the high level due to the good of the soul. By an exact discernment of these goods, we should outline its proper part for the divine element, and for nature. They themselves do not, observe this relation of dignity from the better to the worse. But we do so, when we say that if the body is the organ of the soul, reason is the guide of the entire soul, the mistress of the body, this tent of the soul and that all the other physical advantages should serve only as instruments to the intellectual activity, if you wish it to be perfect in power, duration and wealth.

20. These are the most important conditions to become a sage: first, you must have received from fate a mind endowed with facility to understand, memory, and industry; you must then from youth up exercise your intelligence by the practice of argumentation, by mathematical studies, and the exact sciences. Then you must study healthful philosophy, after which you may undertake the knowledge of the Gods, of laws, and of human life. For there are two means of arriving at this state known as wisdom. The first is to acquire the habit of work that, is intellectual, and the taste for knowledge; the other is to seek to see many things, to undertake business frequently, and to know them either directly at first hand, or indirectly. For he who from youth up has exercised reason by dialectic reasonings, mathematical studies, and exact sciences; is not yet ready for wisdom, any more than he who has neglected these labors, and has only listened to others, and has plunged himself in business. The one has become blind; when the business is to judge particular facts; the other, when he is to judge of general deductions. Just as in calculations you obtain the total by combining the parts, so also, in business practice, reason can vaguely sketch the general formula; but experience alone can enable us to grasp the details and individual facts.

21. Age is in the same relation to youth. Youth makes men energetic, age makes them prudent; never by imprudence does it let a thought escape; it reflects on what it has done; it considers maturely what it ought to do, in order that this comparison of the future with the present, and the present with the future lead it to good conduct. To the past, it applies memory, to the present, sensation, and to the future, foresight; for our memory has always as object the past, foresight the future, and sensation the present. He therefore who wishes to lead an honest and

SECTION IV

POLITICAL FRAGMENTS

22.a. The laws of the wicked and atheists are opposed by the unwritten laws of the Gods, who inflict evils and terrible punishments on the disobedient. It is these divine laws which have developed and directed the laws and written maxims given to men.

b. The relation of law to the soul and human life is identical to that of harmony to the sense of hearing, and the voice; for the law instructs the soul, and therethrough, the life; as harmony regulates the voice through education of the ear. In my opinion, every society is composed of the commander, the commanded, and the laws. Among the latter, one is living; namely the king; the other is inanimate, the written letter. The law is therefore the essential; through it only is the king legitimate, the magistrate, regularly instituted, the commanded free, and the whole community happy. When it is violated, the king is no more than a tyrant; the magistrate illegitimate, the commanded becomes a slave, and the whole community becomes unhappy. Human acts are like a mingled tissue, formed of command, duty, obedience, and force sufficient to overcome resistance. Essentially, the command belongs to the better; being commanded to the inferior, and force belongs to both; for the reasonable part of the soul commands and the irrational part is commanded; both have the force to conquer the passions. Virtue is born from the harmonious cooperation of both; and leads the soul to rest and indifference by turning it away from pleasures and sorrows.

c. Law must conform to nature, and exercise an efficient power over things, and be useful to the political community; for if it lacks one, two, or all of these characteristics, it is no longer a law, or at least it is no longer a perfect law. It conforms to nature if it is the image of natural right; which fits itself, and distributes to each according to his deserts; it prevails, if it harmonizes with the men who are to be subject thereto; for there are may people who are not apt to receive what by nature is the first of goods; and who are fitted to practice only the good which is in relation with them, and possible for them; for that is how the sick and the suffering have to be nursed. Law is useful to the political society if it is not monarchical, if it does not constitute privileged classes, if it is made in the interest of all, and is equally imposed on all. Law must also regard the country and the lands, for not all soils can yield the same returns, neither all human souls the same virtues. That is why some establish the aristocratic constitution, while others prefer the democratic or oligarchic. The aristocratic constitution is founded on the subcontrary proportion, and is the justest, for this proportion attributes the greatest results to the greatest terms, and the smallest to the smallest. The democratic constitution is founded on the geometrical proportion, in which the results of the great and small are equal. The oligarchic and tyrannic constitutions are founded on the arithmetical proportion, which, being the opposite of the subcontrary, attributes to the smallest terms the greatest results, and vice versa.

Such are the kinds of proportions, and you can observe their image in families and political constitutions; for either the honors, punishments and virtues are equally attributed to the great and small, or they are so attributed unequally, according to superiority, in virtue, wealth or power. Equal distribution is the characteristic of democracy; and the unequal, that of aristocracy and oligarchy.

d. The best law and constitution must be a composite of all other constitutions, and contain something democratic, oligarchic, monarchic and aristocratic, as in Lacedemon; for in it the kings formed the monarchic element, the elders the aristocracy, the magistrates the oligarchy, the cavalry generals and youths the democracy. Law must therefore not only be beautiful and good, but its different parts must mutually compensate. This will give it power and durability and by this mutual opposition I mean that the same magistracy command and be commanded, as in the wise laws of Lacedemon; for the power of its kings is balanced by the magistrates, this by the elders, and between these two powers are the cavalry generals and the youths, who, as soon as they see any one party acquire the preponderance, throw themselves on the other side.

The law's first duty is to decide about the gods, the geniuses, the parents; in short, on all that is estimable and worthy; later, about utility. It is proper that the secondary regulations should follow the best, and that the laws be inscribed, not on the houses and doors, but in the depths of the souls of the citizens. Even in Lacedemon, which has excellent laws, the State is not administered by manifold written, ordinances. Law is useful to the political community, if it is not monarchical, and does not serve private interests, if it is useful to all, if it extends its obligation to all, and aims its punishments to shame the guilty, and to brand him with infamy, rather than to deprive him of his wealth. If, indeed, you are seeking to punish the guilty by ignominy, the citizens will try to lead a wiser and more honest life, so as to avoid the law's punishment; if it is only by money fines, they will rate above everything wealth, understanding that it is their best means to repair their faults. The best would be that the State should be organized in a manner such that it would need nothing from strangers, neither for virtue, power,

or anything else. Just as the right constitution of a body, a house, or an army is to contain, and not to depend on outside sources for the principle of its safety; for in that way the body is more vigorous, the house better ordered, and the army will be neither mercenary nor badly drilled. Beings that are thus organized are superior to others; they are free, enfranchised from servitude, unless, for their conservation, they need many things, but have only few needs, easily satisfied. In that way the vigorous man becomes able to bear heavy burdens, and the athlete, to resist cold; for men are exercised by events and misfortunes. The temperate man, who has tested his body and soul, finds any food, drink, even a bed of leaves, delectable. He who has preferred to live like a sybarite among delights, would finally scorn and reject the magnificence of the great (Persian) king. Law must therefore deeply penetrate into the souls and habits of the citizens; it will make them satisfied with their fate, and distribute his deserts to each. Thus the sum, in traversing the zodiac, distributes to everything on the earth, growth, food, life, in the proper measure, and institutes this wise legislation which regulates the succession of the seasons. That is why we call Jupiter *nomios*, law-giver, from *Nemeios*, and we call *nomeus* he who distributes their food to the sheep; that is why we call *nomoi* the verses sung by the citharedians, for these verses impart order to the soul because they are sung according to the laws of harmony, rhythm and measure.

23. The true chief must not only possess the science and power of commanding well, but he must also love men; for it is absurd that a shepherd should hate his flock, and feel hostile disposition towards those he is educating. Besides he must be legitimate; only thus can he sustain a chief's dignity. His science will permit him to discern well, his power to punish, his kindness to be beneficent, and the law to do everything according to reason. The best chief would be he who would closest approach the law, for he would never act in his own interest, and always in that or others, since the law does not exist for itself, but for its subjects.

24. See 21a.

25.When the art of reflexion was discovered, diminished dissensions, and increased concord; those who possess it feel the pride of predominance yielding to the sentiment of equality.

It is by reflexion that we succeed in adjusting our affairs in a friendly fashion; through it the poor receive riches, and the rich give to the poor, each possessing the confidence that he possesses, the equality of rights.

26. Reflexion is like a rule which hinders and turns aside the people who know how to reflect from committing injustices, for it convinces them that they cannot remain hidden if they carry out their purposes and the punishment which has overtaken those who have not known how to abstain makes them reflect and not become back-sliders.

SECTION V

LOGICAL FRAGMENTS

27. Logic, compared with the other sciences is by far the most successful, and succeeds in demonstrating its objectives even better than geometry. Where geometric demonstration fails, the logical succeeds; and logic treats not only with general classes, but with their exceptions.

28. In my opinion it is a complete error to insist that about every subject there are two contrary opinions which are equally true. To begin with, I consider it impossible that, if both opinions are true, they should contradict each other, and that beauty should contradict beauty, and whiteness, whiteness. It cannot be so, for beauty and ugliness, whiteness and blackness are contraries. Likewise, the true is contrary to the false, and you cannot produce two contrary opinions, either true of false; the one must be true, at the expense of the falseness of the other. For instance, he who praises the soul of man and accuses his body is not speaking of the same object, unless you claim that speaking exclusive of the heaven you are speaking exclusively of the earth. Why no, they are not one, but two propositions. What am I trying to demonstrate? That he who says that the Athenians are skillful and witty and he who says they are not grateful, are not supporting contradictory propositions, for contradictories are opposed to each other on the same points, and here the two points are different.

29. ARCHYTAS'S TEN UNIVERSAL NOTIONS. First, all kinds of arts deal with five things: the matter, the instrument, the part, the definition, the end.

The first notion, the substance, is something self-existent and self-subsistent. It needs nothing else for its essence, though subject to growth, if it happens to be something that is born; for only the divine is uncreated, and veritably self-subsistent; for the other notions are considered in relation to substance when the latter by opposition to them is termed self-subsisting: but it is not such, in relation to the divine.

The nine notions appear and disappear without implying the ruin of the subject, the substrate, and that is what is called the universal accident. For the same subject does not lose its identity by being increased or diminished in quantity. Thus, excessive feeding creates excessive size and stoutness; sobriety and abstinence make men lean, but it always the same body, the same substrate. Thus also human beings passing from childhood to youth remain the same in substance, differing only in quantity. Without changing essence, the identical object may become white or black, changing only as regards quality. Again, without changing essence, the identical man may change disposition and relation, as he is friend or enemy; and being today in Thebes, and tomorrow in Athens changes nothing in his substantial nature. Without changing essence, we remain the same today that we were yesterday; the change affected only time; the man standing is the same as the man sitting; he has changed only in situation: Being aimed or unarmed is a difference only of possession; the striker and the cutter are the same man in essence, though not in action; he who is cut or struck Ñ which belongs to the category of suffering, Ñ still retains his essence. The differences of the other categories are clearer; those of quality, possession, and suffering present some difficulties in the differences; for we hesitate about the question of knowing if having fever, shivering or rejoicing belong to the category of quality, possession, or

suffering. We must distinguish: if we say, it is fever, it is shivering, it is joy, it is quality; if we say, he has fever, he shivers, he rejoices, it is possession; while possession again differs from suffering, in that the latter can be conceived without the agent. Suffering is a relation to the agent, and is understood only by him who produces it; if we say, he is cut, he is beaten, we express the patient; if we say, he suffers, we express possession.

We say that (Archytas) has ten, and no more universal notions; of which we may convince ourselves by the following division: the being is in a subject, (a substance), or is not in a subject; that which is not in a subject, forms the substance; that which is in a subject or is conceived by itself, or is not conceived by itself; that which is not conceived by itself constitutes relation, for relative beings, which are not conceived by themselves, but which forcibly import the idea of an other being, are what is called scheseis, conditions. Thus the term son is associated with the term farther, that of slave, master; thus all relative beings are conceived in a necessary bond together with something else, and not by themselves. The self-conceivable being is either divisible Ñ when it is quantity, --- or indivisible, when it constitutes quality. The six other notions are produced by combination of the former. Substance mingled with quantity, if seen in place, constitutes the category of *where*; if seen in time, constitutes that of *when*. Mingled with quality, substance is either active and forms the category of *action*, or when passive, forms that of *suffering*, or, *passivity*. Combined with relation, it is either posited in another, and that is what is called *situation*, or it is attributed to somebody else, and then it is *possession*.

As to the order of the categories, quantity follows substance and precedes quality; because, by a natural law, everything that receives quality also receives mass, and that it is only of something so determinate that quality can be so affirmed and expressed. Again, quality precedes relation, because the former is self-sufficient, and the latter by a relation; we first have to conceive and express something by itself before in a relation.

After these universal categories follow the others. Action precedes passivity, because its force is greater; the category of situation precedes that of possession, because being situated is something simpler than being possessed; and you cannot conceive something attributed to another without conceiving the former as situated somewhere. That which is situated is also in a position, such as standing, seated or lying. The characteristic of substance is more or less-ness; for we say that a man is no more of an animal than a horse, by substance, and not to admit the contraries. The characteristic of quality is to admit more or less; for we say, more or less white, or black. The characteristic of; quantity is to admit equality or inequality; for a square foot is not equal to an acre, and 144 sq. inches equal a square foot; five is not equal to ten, and twice five is equal to ten. The characteristic of relation is to join contraries; for if there is a father, there is a son; and if there is a master there is a slave. The characteristic of whereness is to include; and of whenness it to remain; of situation, to be located; of possession, to be attributed. The composite of substance and quantity is anterior to the composite of quality; the composite of substance and quality in its turn precedes that of substance and relation. Whereness precedes whenness; because whereness presupposes the place that is fixed and permanent; whenness relates to time and time, ever in movement, has no fixity; and rest is anterior to movement. Action is anterior to passivity, and situation to possession.

1. CATEGORY OF SUBSTANCE. Substance is divided into corporeal and incorporeal; the corporeal into bodies animate and inanimate. Animated bodies, into those endowed with sensation, and without sensation. Senses-bodies into animals and zoophytes, which do not farther divide into opposite distinctions. The animal is divided into rational and irrational; the rational into mortal and immortal; the mortal into differences of genus, such as man, ox, horse, and the rest. The species are divided into individuals who have no abiding value. Each of the sections that we obtained above by opposite divisions is susceptible of being in turn divided equally, until we arrive to the indivisible individuals who are of no value.

2. CATEGORY OF QUANTITY. This is divided into seven parts: the line, surface, the body, the place, the time, the number, and language. Quantity is either continuous or discrete; of continuous qualities there are five; of the discrete, number and language. In quantity, you may distinguish that which is composed of parts having position relative to each other; such as line, surface, body and place; and those whose parts have no position, such as number, language, and time; for although time is a continuous quantity, nevertheless its parts have no position; because it is not permanent, and that which has no permanence could not have any position. Quantity has produced four sciences: immovable quantity, geometry; movable continuous quantity, astronomy; immovable discrete quantity, arithmetic; and the movable, music.

3. CATEGORY OF QUALITY. This is divided into hexis or habit; and diathesis or affection; passive quality and passivity, power and impotence, figure and form. Habit is affection in a state of energetic tension; it is the permanence and fixity derived from continuity and the energy of affection; it is affection become (second) nature, a second enriched nature. Another explanation of habit is the qualities given us by nature, and which are derived neither from affection, nor from the natural progress of the being; as sight and the other senses. Both passive quality and passivity are increase, intensity, and weakening. To both of these are attributed anger, hate, intemperance, the other vicious passions, the affections of sickness, heat and cold; but these are classified at will under habit and affection, or under passive quality and passivity. You might say that so far as affection is communicable it might be called habit; so far as it cause a passion, it might be called a passive quality; which refers both to its permanence and fixity. For a modification contained in the measure is called passion. Thus from the one to whom it is communicated, heat may be called a habit; from the cause which produces the modification; we may say that it is either the passive quality, or the power of the passion; as when we say of a child that he is potentially a runner or a philosopher, and, in short, when at a given moment the being does not have the power to act, but that it is possible that after the lapse of a certain period of time, this power may belong to him. Impotence is when nature refuses itself to the possibility of accomplishing certain actions as when the man is impotent to fly, the horse to speak, the eagle to live in water, and all the natural impossibilities.

Figure is a conformation of a determined character; form, the quality showing itself exteriorly by color, or beauty, or ugliness showing itself on the surface by color, and in short any form that is apparent, determinate and striking. Some limit figure to inanimate things; reserving form to living beings. Some say that the word figure gives the idea of the dimension of depth; and that the form is applied only to the superficial appearance; but you have been taught all of that.

4. CATEGORIES OF RELATION. Generally, the relatives are divided into four classes; nature, art, chance and will. The relation of father to son is natural; that of master to disciple, that of art; that of master to slave, that of chance; and that of friend to friend, and enemy to enemy, that of will; although you might say that these are all natural relations.

5. CATEGORY OF WHERENESS. The simplest division is into six: up, down, forwards, backwards, right and left. Each of these' subdivisions contains varieties. There are many differences in up-ness, in the air, in the stars, to the pole, beyond the pole; and such differences are repeated below; the infinitely divided places themselves are further subject to an infinity of differences, but this very ambiguous point will be explained later.

6. CATEGORY OF WHENNESS. This is divided in present, past and future; the present is indivisible, the past is divided into nine subdivisions, the future into five; we have already spoken of them.

7. CATEGORY OF ACTION. This is divided into action, discourse and thought; action in work of the hands, with tools, and with feet; and each of' these divisions is subdivided into technical divisions which also have their parts. Language is divided into Greek, barbarian, and each of these divisions have their parts, namely, its dialects. Thought is divided into and infinite world of thoughts, whose objects are the worlds, other people, and the hypercosmic. Language and thought really belong to action, for they are the acts of the reasonable nature; in fact, if we are asked: What is Mr. X doing? we answer, he is chatting, conversing, thinking, reflecting, and so on.

8. CATEGORY OF PASSIVITY. Passivity is divided in suffering of the soul, and of the body; and each of these is subdivided into passions which result from actions of somebody else, as for instance, when somebody is struck; and passions which arise without the active intervention of someone else; which occur in a thousand different forms.

9. CATEGORY OF SITUATION. This is divided into three: standing, sitting, and lying; and each of these is subdivided by differences of location. We may stand on our feet, or on the tips of our fingers; with the leg unflexed, or the knee bent; further differences are equal or unequal steps; or walking on one or two feet. Being seated has the same differences; one may be straight, bent, reversed; the knees may form an acute or obtuse angle; the feet may be placed over each other, or in some other way. Likewise with lying down; prone or head forwards, or to the side, the body extended, in a circle, or angularly. Far from uniform are these divisions; they are very various. Position is also subject to other divisions, for an instance, an object may be spread out like corn, sand, oil, and all the other solids; that are susceptible of position, and all the liquids that we know. Nevertheless being extended belongs to position, as cloth and nets.

10. CATEGORY OF POSSESIONS. "Having" signifies things that we put on, as shoes, arms, coverings; things which are put on others, such as a peck, a bottle, and other vases; for we say that the peck has oats, that the bottle has wine; also of wealth and estates; we say, he has a fortune, fields, cattle and other similar things.

30. The order of the categories is the following; in the first rank is substance, because it alone serves as substrate to all the others, that we can conceive it alone, and by itself, and that the others cannot be conceived without it; for all attributes' subject reside therein, or are affirmed thereof. The second is quality; for it is impossible for a thing to have a quality without an essence.

31. Every naturally physical and sensible substance must, to be conceived by man, be either classified within the categories, or be determined by them, and cannot be conceived without them.

32. Substance has three differences; the one consists in matter, the other in form, and the third in the mixture of both.

33. These notions, these categories, have characteristics that are common and individual. I say that they are characteristics common to substance, not to receive more or lessness; for it is not possible to be more or less man, God or ant; to have no contraries for man is not the contrary of man, neither god of a god; neither is it contrary to other substances, to exist by oneself, and not to be in another, as green or blue color is the characteristic of the eye, since all substance depends on itself. All the things that belong to it intimately, or the accidents are in it; or cannot exit without it;....quality is suited by several characteristics of substance, for example, not to be subject to more or lessness.

34. It is the property to remain self-identical, one in number, and to be susceptible of the contraries. Waking is the contrary of sleep; slowness to swiftness, sickness to health; and the same man, identically one, is susceptible of all these differences. For he awakes, sleeps, moves slowly or quickly, is well or sick, and in short is able to receive all similar contraries, so long as they be not simultaneous.

35. Quantity has three differences: one consists in weight, like bullion; the other in size, as the yard; the other in multitude, as ten.

36. Including its accidents, substance is necessarily primary; that is how they are in relation to some thing else; after the substance come the relations of accidental qualities.

37. A common property which must be added to quality, is to admit certain contraries, and privation. The relation is subject to more or lessness; for though a being remain ever the same, to be greater or smaller than anything else is moreness; but all the relations are not susceptible thereto; for you cannot be more or less father, brother, or son; whereby I do not mean to express the sentiments of both parents, nor the degree of tenderness is held mutually by beings of the same blood, and the sons of the same parents; I only mean the tenderness which is in the nature of these relations.

38. Quality has certain common characters, for example, of receiving the contraries, and privation; more and less affect the passions. That is why the passions are marked by the characteristics of indetermination, because they are in a greater or less indeterminate measure.

39. Relation is susceptible of conversion, and this conversion is founded either on resemblance, as the equal, and the brother, or on lack of resemblance, the large and the small. There are relatives which are not converted, for instance, science and sensation; for we may speak of the science of the intelligible, and of the sensation of the sensible; and the reason is that the intelligible and the sensible can exist independently of science and sensation; while science and sensation cannot exist without the intelligible and the sensible;.....The characteristic of relatives is to exist simultaneously in each other; for if we grant the existence of doubleness, the half must necessarily exist; and if the half exists; necessarily must the double exist, as it is the cause of the half, as the half is the cause of thy double.

40. Since every moved thing moves in a place, since action and passivity are actualized movements, it is clear that there must be a primary place in which exist the acting and the passive objects.

41. The characteristic of the agent is to contain the cause of the motion; while the characteristic of the thing done, which is passive, is to have it in some other. For the sculptor obtains the cause of the making of the statue, the bronze possesses the cause of the modification it undergoes, both in itself and in the sculptor. So also with the passions of the soul, for it is in the nature of anger to be aroused as the result of something else; that it be excited by some other external thing, for example, by scorn, dishonor, and outrage; and he who acts thus towards another, contains the cause of his action.

42. The highest degree of the action, is the act; which contains three differences; it may be accomplished in the contemplation of the stars, or in doing, such as healing or constructing; or in action, as in commanding an army, or in administering the affairs of state. An act may occur even without reasoning, as in irrational animal. These are the most general contraries.

43. Passion differs from the passive state; [as] passion is accompanied by sensation, like anger, pleasure and fear; while one can undergo something without sensation, such as the wax that melts, or the mud that dries. Then also the [deed] done differs also from the passive state, when the deed done has undergone a certain action, [whi]le everything that has undergone a certain action is not a deed done; for a thing may be in a passive state as a result of lack of privation.

44. On the one side there is the agent, on the other, the patient; for example, in nature, God is the being who acts; matter is the being which undergoes, and the elements are neither the one nor the other.

45. The characteristic of possession is to be something adventitious, something corporeal, separated from essence. Thus a veil or shoes, are distinct from the possessor; those are not natural characteristics, nor essential accidents, like the blue color of the eyes, and rarefaction; these are two incorporeal characteristics, while possession relates to something corporeal and adventitious.

46. Since the signs and the things signified have a purpose, and since the man who uses these signs and signified things is to fulfill the perfect function of speech, let us finish what we have said by proving that the harmonious grouping of all these categories does not belong to man in

general, but to a certain finite individual. Necessarily, it must be a definite man existing somewhere who possesses quality, quantity, relation, action, passivity, location and possession, who is in a place and time. The man in himself receives only the first of these expressions; I mean essence and form; but he has no quality, no age, he is not old, he neither does, nor suffers anything, he has no location, he possesses nothing, he is neither in place nor time. All those are only accidents of the physical and corporeal being; but not of the intelligible, immovable, and indivisible being.

47. Among contraries, some are said to be mutually opposed by convention and nature, as good to evil, the sick to the well man, truth to error [and] others, as possession is opposed to privation, such as life and death, sight and blindness, science and ignorance; others as relatives as the double and the half, the commander and the commanded, the master and the slave; others, like affirmation and negation, as being man and not a man, being honest, and not.

48. The relatives arise and disappear necessarily simultaneously; the existence of the double is impossible, without implying that of the half and vice versa. If something becomes double, the half must arise, and if the double is destroyed, the half passes away with it.

49. Of the relatives, some respond to each other in two senses; as, the greater, the smaller, the brother, the relative. Others again respond, but not in the two senses, for we say equally, the science of the intelligible, and the science of the sensible, but we do not say the reciprocal, the intelligible of science, and the sensible of sensation. The reason is that the object of judgment can exist independently of him who judges, for instance, the sensible can exist without sensation, and the intelligible without science; while it is not possible that the subject which bears a judgment exists without the object which he judges; for example, there can be no sensation without sensible object, nor science without intelligible object. Relatives which respond reciprocally are of two kinds; those that respond indifferently, as the relative, the brother, the equal; for they are mutually similar, and equal. Some respond reciprocally, but not in-differently; for this one is greater than that one, and that one is smaller than this one; and this one is the father of that one, and that one the son of this one.

50. These opposites divide into kinds which hang together; for of the contraries, some are without middle term, and the others have one. There is no middle term between sickness and health, rest and movement, waking and sleep, straightness and curvedness, and such other contraries. But between the much and the little, there is a just medium; between the shrill and the low, there is the unison; between the rapid and slow, there is the equality of movement; between the greatest and the smallest, the equality of measure. Of universal contraries there must be one that belongs to what receives them; for they do not admit any medium term. Thus there is no medium term between health and sickness; every living being is necessarily sick or well; neither between waking and sleeping, for every living being is either awake or asleep; nor between rest and movement, for every human being is either at rest or at movement. The opposites of which neither of any or both necessarily belong to the subject that may receive them; have any middle terms; between black and white, there is the [----]; and it is not necessary that an animal be black or white; between the great and the small, there is the equal; and it is not necessary that a living being be either great or small; between the rough and the soft, there is the gentle, and it is not necessary that a living being be either rough or soft. In the opposites there are three differences:

some are opposed, as the good is to evil, for instance, health to sickness; the others like evil to evil, as for instance, avarice to lechery; the others, as being neither the one or the other for instance, as white is opposed to black, and the heavy to the light. Of the opposites, some occur in the genus of genera; for the good is opposed to evil, and the good is the genus of virtues, and evil that of the evils. Others occur in the genera of species, virtue is the opposite of vice and virtue is the genus of prudence and temperance, and vice is the genus of foolishness and debauch. Others occur in the species, courage is opposed to cowardliness, justice of injustice, and justice and virtue are species of virtue, injustice and debauch species of vice. The primary genera, which we call genera of genera, can be divided; the last species, which are the immediate nearest to the object, that is sensible, could no longer be genera, and are only species. For the triangle is the genus of the rectangle, of the equilateral and of the scalene...the species of good.....

51. The opposites differ from each other in that for some, the contraries, it is not necessary that they arise at the same time, and disappear simultaneously. For health is the contrary of sickness, and rest that of movement; nevertheless neither of them arises or perishes at the same time as its opposite. Possession and privation of production differ in this, that it is in the nature of contraries that one passes from one to the other, for instance, from sickness to health and vice versa. It is not so with possession and privation; you do indeed pass from possession to privation, but the privation does; not return to possession; the living, die, but the dead never return to life. In short, possession is the pesistance of what is according to nature, while privation is its lack, and decay. Relatives necessarily arise and disappear simultaneously; for it is impossible for the double to exist without implying the half ; or vice versa. If some double happens to arise, it, is impossibly that the half should not arise, or if some double be destroyed, that the half be not destroyed. Affirmation and negation are forms of preposition, and they eminently express the true and the false. Being a man is a true proposition, if the thing exists, and false if it does not exist. You could say as much of negation, it is true or false according to the thing expressed.

Besides, between good and evil there is medium, which is neither good nor evil; between much and little, the just measure; between the slow and the fast, the equality of speed; between possession and privation, there is no medium. For there is nothing between life and death; between sight and blindness; unless indeed you say that the living who is not. yet born, but who is being born, is between life and death, and that the puppy who does not yet see is between blindness and sight. In such an expression, we are using an accidental medium, and not according to the true and proper definition of contraries.

Relatives have middle terms; for between the master and the slave, there is the free man, and between the greatest and the smallest, there is equality; between the wide and the narrow there is the proper width; one might likewise find be between the other contraries a medium, whether or no it has a name.

Between affimation (and negation) there are no contraries, for instance, between being a man and not being a man, being a musician and not being a musician. In short, we have to affirm or deny.

Affirming is showing of something that it is a man, for instance, or a horse, or an attribute of these beings, as of the man that he is a musician, and of the horse, that he is warlike; we call denying, when we show of something that it is not something, not man, not horse, or that it lacks

an attribute of these beings, for instance, that the man is not a musician and that the horse is not warlike; and between this affirmation and this negation, there is nothing.

52. Privation, and being deprived is taken in three senses; or one does not at all have at all have the thing, as that the blind man does not have sight, the mute does not have voice, and the ignorant, no science; or that one does not have it but partially, as that the man hard of hearing has hearing, and that the man with sore eyes has sight; or one can say that partially he does not have it, as one says that a man whose legs are crooked that he has no legs, and of a man who has a bad voice, that he has no voice.

Aristoxenus

Aristoxenus of Tarentum (4th century BC) was a Greek peripatetic philosopher, and writer on music and rhythm.

He was taught first by his father Spintharus, a pupil of Socrates, and later by the Pythagoreans, Lamprus of Erythrae and Xenophilus, from whom he learned the theory of music. Finally he studied under Aristotle at Athens, and was deeply annoyed, it is said, when Theophrastus was appointed head of the school on Aristotle's death.

His writings, said to have numbered four hundred and fifty-three, were in the style of Aristotle, and dealt with philosophy, ethics and music. The empirical tendency of his thought is shown in his theory that the soul is related to the body as harmony to the parts of a musical instrument. We have no evidence as to the method by which he deduced this theory (cf. Theodor Gomperz, *Greek Thinkers*, Eng. trans. 1905, vol. iii. p. 43).

In music he held that the notes of the scale are to be judged, not as the Pythagoreans held, by mathematical ratio, but by the ear. The only work of his that has come down to us is the three books of the *Elements of Harmony*, an incomplete musical treatise. Grenfell and Hunt's *Oxyrhynchus Papyri* (vol. i., 1898) contains a five-column fragment of a treatise on metre; probably this treatise of Aristoxenus.

BIOGRAPHY OF OCELLUS LUCANUS

Practically nothing is known of the life of Ocellus, except that Iamblichus mentions the name of his brother Ocillus, and his sister Byndacis, all Pythagorean philosophers. In the biography of Archytas we read his writings were preserved by his family, so we may assume he returned home, after studying Pythagoreanism.

His significance, however, is great, for those letters of Plato witness how much he sought them, and that he indeed received some of them. Of the books that we have, Philo Judaeus reedited the first, in his writing *on the Incorruptibility of the World*. The second was used, almost word for word by Aristotle, in his tract *on Generation and Corruption*; and the fourth was used word for word by Iamblichus in his Life of Pythagoras. Ocellus was therefore much appreciated, and a very useful writer.

In his way, Archytas was almost as useful to Aristotle, in fragments: 6, 8, 1, 11, 17 (4), (5), (8); 9 (2), (4), (9), (10); 32; etc....

The truth is that Pythagoreanism was bodily adapted by Plato and Aristotle, who thereby made their fortunes. Pythagoreanism was an unselfish inspiration; and not until these fragments are united has it been possible to pass through Plato and Aristotle, to the real spring of Greek philosophy. As an instance, Plato wrote his Timaeus as an amplification of the book of the Pythagorean's [Locrian] Timaeus's tract which has been preserved along with Plato's works.

I

THE PYTHAGOREAN'S TREATISE ON THE UNIVERSE

Ocellus Lucanus has written what follows concerning the nature of the universe; having learnt some things through clear arguments from nature herself, but others from opinion, in conjunction with reason, it being his intention (he) to derive what is probable from intellectual perception. Therefore it appears to me, that the universe is indestructible and unbegotten, since it always was, and always will be; for if it had a temporal beginning, it would not always have existed: thus therefore, the universe is unbegotten and indestructible; for if someone should opine that it was once generated, he would not be able to find anything into which it can be corrupted arid dissolved, since that from which it was generated would be the first part of the universe; and again, that into which it would be dissolved would be the last part of it.

But if the universe was generated, it was generated together with all things; and if it should be corrupted, it would be corrupted together with all things. This however is impossible. This universe is therefore without a beginning, and without an end: nor is it possible that it can have any other mode of subsistence.

To which may be added, that everything which. has received a beginning of generation, and which ought also to participate of dissolution, receive two mutations; one of which, indeed proceeds from the less to the greater, and from the worse to the better; and that from which it begins change is denominated generation, but that a which its length it arrives at is called climax.

The other mutation, however, proceeds from the greater to the less, and from the better to the worse; but the end of this mutation is called corruption and dissolution.

If therefore the whole and the universe were generated, and are corruptible they must, when generated have been changed from the less to the greater, and from the worse to the better; but when corrupted, they must be changed from the greater to the less, and from the better to the worse. Hence, if the world was generated, it would receive increase and would arrive at its consummation; and again, it would afterwards decrease and end. For every thing which has a progression possesses three boundaries, and two intervals; the three boundaries are generation, consummation and end; and the intervals are, progression from generation to consummation, and from consolation to end.

The whole, however, and the universe, affords as from itself, no indication of anything of this kind; for neither do we perceive it rising into existence, or becoming to be, nor changing to the better and the greater, nor changing to worse or less; but it always continues to subsist in identical manner, and perpetually remains self-identical.

Clear signs and indications of this are the orders of things, their symmetry, figurations, positions, intervals, powers, swiftness and slowness in respect to each other; and, besides these, their numbers and temporal periods, are clear signs and indications. For all such things as these change and diminish, conformably to the course of generation; for things that are greater and better tend towards consummation through power, but those that are less and worse decay through the inherent weakness of nature.

The whole world is what I call the whole universe; for this word "cosmos" was given it as a result of its being adorned with all things. From itself it is a consummate and perfect system of all things, for there is nothing external to the universe, since whatever exists is contained in the universe, and the universe subsists together with this, comprehending in itself all things, both parts and superfluous.

The things contained in the world are naturally congruous with it; but the world harmonizes with nothing else, symphonizing with itself. Other things do not possess self-sub sistence, but require adjustment with the environment. Thus animals require conjunction with air for the purpose of respiration; and with light, in order to see; and similarly the other senses with other environment, to function satisfactorily. A conjunction with earth is necessary for the germination of plants. The sun, moon, planets and fixed stars likewise integrate with the world, as parts of its general arrangement. The world, however, has no conjunction with any thing outside of itself. The above is supported by the following. Fire which imparts heat to others, is self-hot; honey which is sweet to the taste, is self-sweet. The principles of demonstrations, which conclude to things unapparent, are self-evident. Therefore the cause of the perfection of other things is itself perfect. That which preserves and renders permanent other things must itself be preserved and permanent. What harmonizes must itself be self-harmonic. Now as the world is the cause of the existence, preservation and perfection of other things, must itself be perpetual and perfect; and because its duration is everlasting, it becomes the cause of the permanence of all other things. In short, if the universe should be dissolved, it would be dissolved either into the existent, or nonexistent. As it could not be dissolved into existence, for in this case the dissolution would not be a corruption;

as being is either the universe, or some part of it. Nor can it be dissolved into nonentity, since being cannot possibly arise from non-being; or be dissolved into nonentity. Therefore the universe is incorruptible, and never can be destroyed. If, however, somebody should think that it can be corrupted, it must be corrupted either from something external to, or contained in the universe, but it cannot be corrupted by anything external to it, for nothing such exists, since all other things are comprehended in the universe, and the world is the whole and the all. Nor can it be corrupted by the things it contains, which would imply their greater power. This however is impossible; for all things are led and governed by the universe, and thereby are preserved and adjusted, possessing life and soul. But if the universe can neither be corrupted by anything externa1 to it, nor by anything contained within it, the world must therefore be incorruptible and indestructible; for we consider the world identical with the universe.

Further, the whole of nature surveyed through its own totality, will be found to derive continuity from the first and most honorable bodies, proportionally attenuating this continuity, introducing it to everything mortal, and receiving the progression of its peculiar subsistence; for the first (and most honorable) bodies in the universe revolve according to the same and similarly. The progression of the whole of nature, however, is not successive and continuous, nor yet local, but is subject to mutation. When condensed, fire generates air; air water, and water earth. A return circuit of transformation extends backward from earth to fire, whence it originated. However fruits, and most rooted plants, originate from seeds. When however they fruit and mature, that are again resolved into seed, nature producing a complete circular progression.

In a subordinate manner, men and other animals change the universal boundary of nature; for in these there is no periodical return to the first age; nor is there a transfusion, such as between fire and air, and water and earth; but the mutations of their ages being accomplished in a four-cycled circle, they are dissolved, and reformed.

These therefore are the signs and indications that the universe which comprehends (all things) which, will always endure and be preserved, but that, its parts, and its nonessential additions are corrupted and dissolved. Further, it is credible that the universe, without a beginning, and without end, from s figure, motion, time and essence; and there, may be concluded that the world is begotten and incorruptible; for its figure is circular; and as a circular figure is similar and equal on all sides, it is therefore without a beginning or an end. Circular is also the motion of the universe, but this motion is stable and without transition. Time, likewise, in which motion exists, infinite; for neither had this a beginning, nor will it have an end of its revolution. The universe's essence also does not waste elsewhere, and is immutable, because it is not naturally adapted to charge, either from worse to better, or from better to worse. From all these arguments, therefore, it is obviously credible, that the world is unbegotten and incorruptible. So much about the world and the universe.

II

CREATION

Since, however, in the universe there is a difference between generation and the generated, and since generation occurs where there is a mutation and egress from things which rank as subjects,

then must the cause of generation subsist as long as the generated matter. The cause of generation must be both efficient and motive, while the recipient must be passive, and moved. The Fates themselves, distinguish and separate the impassive part of the world from that which is perpetually in motion. For the course of the moon is the meeting-line of generation and immortality. The region above the moon, as well as the lunar domain, is the residence of the divinities; while sub-lunar regions are the abode of strife and nature; here is change of the generated things, and regeneration of those that have perished.

So that part of the world, however, in which nature and generation predominate, it is necessary that the three following things be present. In the first place, the body which yields to the touch, and which is the subject of all generated natures. But this will be an universal recipient, a characteristic of generation itself, having the same relation to the things that are generated from it, as water to taste, silence to sound, darkness to light, and the matter of artificial forms to the forms themselves. For water is tasteless and devoid of quality, yet is capable of receiving the sweet and the bitter, the tart and the salt. Air also, which is formless as regards sound, is the recipient of words and melody. Darkness, which is without color, and without form, becomes the recipient of splendor, and of the yellow color, and the white; but white pertains to the statuary's art, and the wax sculptor's art. Matter's relation, however, is different from the sculptor's art, for in matter, prior to generation, all things are in capacity, but they exist in perfection when they are generated, and receive their proper nature. Hence matter (or a universal recipient) is necessary to the existence of generation.

The second necessity is the existence of contrarieties, in order to effect mutations and changes in quality, matter, for this purpose, receiving passive qualities, and an aptitude to [the] participations of forms. Contrariety is also necessary in order that powers which are naturally mutually repugnant may not finally conquer, or vanquish each other. These powers are hot and cold, dryness and moistness.

In the third place rank essences: and those and fire and water, air and earth, of which heat and cold, dryness and moistness, are powers. But essences differ from powers, essences being locally corrupted or generated, as their reasons or forms are incorporeal.

Of those four powers, however, heat and cold subsist as causes and things of an effective nature; but the dry and the moist rank as matter and things that are passive, though matter is the first recipient of things, for it is that which is spread under all things in common. Hence the body, whose capacity is the object of sense, and ranks as a principle, is the first thing while contraries, such as heat and cold, moistness and dryness, rank as primary differences; but heaviness and lightness, density and rarity, are related as things produced from primary differences. These amount to sixteen: heat an cold, moistness and dryness, heaviness and lightness, rarity and density, smoothness and roughness, hardness and softness, thinness and thickness, acuteness and obtuseness. Knowledge of all of these is had by touch, which forms a judgment; hence also any body whatever which contains capacity for these can be apprehended by touch.

Heat and dryness, rarity and sharpness are the powers of fire; coldness and moistness, density and obtuseness are those of water; those of air are softness, smoothness, light, and the quality of being attenuated; while those of earth are hardness and roughness, heaviness and thickness.

Of these four bodies, however fire and earth are the intensities of contraries. Fire is the intensity of heat, as ice is of cold; and if ice is a concretion of moisture and farigidity, fire will be the fervor of dryness and heat. That is why neither fire nor ice generate anything.

Fire and earth, therefore, are the extremities of the elements, while water and air are the media, for they have a mixed corporeal nature. Nor is it possible that there could be only one of the extremes, a contrary thereto being necessary. Nor could there be two only, for it is necessary to have a medium, as media oppose extremes.

Fire therefore is hot and dry, but air is hot and moist; water is moist and cold, and earth cold and dry. Hence heat is common to air and fire; cold is common to water and earth; dryness to earth and fire, and moisture to water and air. But with respect to the peculiarities of each, heat is the peculiarity of fire, dryness of earth, moisture of air, and frigidity of water. These essences remain permanent, through the possession of common properties; but they change through such as are peculiar, when one contrary overcomes another.

Hence, when the moisture in air overcomes the dryness in fire, or when water's frigidity overcomes air's heat, and earth's dryness water's moistness, and vice versa, then are effected the mutual mutations and generations of the elements.

The body, however, which is the subject and recipient of mutations, is a universal receptacle, and is in capacity the first tangible substance. But the mutations of the elements are effected either from a change of earth into fire, or from fire into air, or from air into water, or from water into earth. Mutations is also effected, in the third place, when each element's contrariness is corrupted, simultaneously with the preservation of everything kindred and coeval. Generation therefore is effected when one contrary quality is corrupted. For fire, indeed, is hot and dry, but air is hot and moist, and heat is common to both; but the peculiarity of fire is dryness, and of air, moisture. Hence when the moisture in air overcomes the dryness in fire, then fire is changed into air.

Again, since water is moist and cold, but air is moist and hot, moisture is common to both. Water's peculiarity is coldness, and of air, heat. When therefore the coldness in water overcomes the heat in air, air is altered into water.

Further, earth is cold and dry, and water cold and moist; coldness being common to both. But earth's peculiarity is dryness, and water's, moisture. When therefore earth's dryness overcomes water's moisture, water is altered into earth.

Earth's mutation in the ascending alteration occurs in a contrary way. One alternate mutation is effected when one whole vanquishes another; and two contrary powers are corrupted, nothing being common to them, at the same time. For since firs is hot and dry, while water is cold and moist, when the moisture in water overcomes the dryness in fire, and water's coldness, fire's heat; then fire is altered into water.

Again, earth is cold and dry, while air is hot and moist. When therefore earth's coldness overcomes air's heat, and earth's dryness air's moisture, then air is altered into earth.

When air's moisture corrupts fire's heat, then from both of them will be generated fire; for air's heat, and fire's dryness will remain, fire being hot and dry.

When earth's coldness is corrupted, and also water's moisture, then from both of them will be generated earth. For earth's dryness and water's coldness will be left, as earth is cold and dry.

But when air's heat and fire's heat are corrupted, no element will be generated; for in both of these will remain contraries, air's moisture and fire's dryness. Moisture is however contrary to dryness.

Again, when earth's coldness, and that of water are corrupted, neither thus will any generation occur; for earth's dryness, and water's moisture will remain. But dryness is contrary to moisture.

Thus we have briefly discussed the generation of the first bodies, and how and from what subjects it is effected.

Since, however, the world is indestructible and unbegotten, and neither had a beginning or generation, nor an end, it is necessary that the nature which produces generation in another thing, and also that which generates in itself, should be simultaneously present. That which produces generation in another thing, is the whole superlunary region; though the more proximate cause is the sun, who by its comings and goings continually changes the air; from cold to heat, which again changes the earth, which alters all its contents.

The obliquity of the zodiac, also, is well placed in respect to the sun's motion, for it likewise is the cause of generation. This is universally accomplished by the universe's proper order; wherein some things are active, and others passive. Different therefore is the generator, which is superlunary, while that which generated is sublunary; and that which consists of both of these, namely, an

III

THE PERPETUITY OF THE WORLD

Man's generation, did not originate from the earth, other animals, or plants; but the world's proper order being perpetual, its contained, aptly arranged natures should share with it never-failing subsistence. As primarily the world existed always, its parts must coexist with it; and by these I mean the heavens, the earth, and what is contained between them; which is on high, and is called aerial; for the world does not exist without, but with and from these.

As the world's parts are con subsistent, their comprehended natures must coexist with them; with the heavens, indeed, the sun, moon, fixed stars and planets; with the earth, animals and plants, gold and silver; with the aerial region, spiritual substances and wind, heating and cooling; for it is the property of the heavens to subsist in conjunction with the natures which it comprehends, and of the earth to support its native plants and animals; of the aerial regions, to be consubsistent with the natures it has generated. Since the therefore in each division of the world there is arranged a certain genus of animals which surpasses its fellows, the heavens are the habitat of the

gods, on the earth men, and in the space between, the geniuses. Therefore the race of men must be perpetual, since reason convinces us that not only are the world's parts consubsitent with it, but also their comprehended natures.

Sudden destructions, and mutations however take place in the parts of the earth; the sea overflows on to the land, or the earth shakes and spits, through the unobserved entrance of wind or water. But an entire destruction of the earth's whole arrangement never took place, nor ever will. Hence the story that Grecian history began with the Argive Inachus is false, if understood to be a first principle, but true, as some mutations of Greek politics; for Greece has frequently been, and will again be barbarous, not only from the irruption of foreigners, but from Nature herself, which, although she does not become greater or less; yet is always younger, and has a beginning in reference to us.

So much about the whole, and the universe; the generation and corruption of natures generated in it; how they subsist, and for ever; one part of the universe consisting of a nature which is perpetually moved, and another passive one; the former governing, the latter ever governed.

IV

GROWTH OF MEN

Law, temperance and piety conspire in explaining as follows the generation of men from each other, after what manner, from what particulars, and how effected. The first postulate is that sexual association should occur never for pleasure but only for procreation of children.

Those powers and instruments, and appetites ministering to copulation were implanted in man by divinity, not for the sake of voluptuousness, but for the perpetuation of the race. Since it was impossible that man, who is born mortal, should participate in a divine life were his race not immortal, divinity operated this immortality through individuals, and lent continuousness to mankind's generation. This is the first essential, that cohabitation should not be effected for mere pleasure.

Next, man should be considered in connection with the social organism, a house or city, and especially that each human progeny should work at the completion of the world, unless he plans to be a deserter of either the domestic, political or divine Vestal hearth.

For those who are not entirely connected with each other for the sake of begetting children, injure the most honorable system of convention. But if persons of this description procreate with libidinous insolence and intemperance, their offspring will be miserable and flagitious, and will be execrated by God and geniuses, by men, families and cities.

Those therefore who deliberately consider these things ought not, in a way similar to irrational animals, to engage in venereal connections, but should think copulation a necessary good. For it is the opinion of worthy men that it is necessary and beautiful, not only to fill houses with large families, and also the greater part of the earth (for man is the most mild and the best of all

animals), but as a thing of the greatest consequence, to cause them to abound with the most excellent men.

For on this account men inhabit cities governed by the best laws, rightly manage their domestic affairs and if they are able, impart to their friends such political employments as are conformable to the polities in which they live, since they not only provide for the multitude at large, but especially for worthy men.

Hence many men err who enter into connubial state without regarding the magnitude off the power of fortune, or public utility, but direct their attention to wealth, or dignity of birth. For in consequence of this, instead of uniting with females who are young and in the flower of their age, they become connected with extremely old women; and instead of having wives with a disposition according with, and most similar to their own, they marry those who are of an illustrious family, or are extremely rich. On this account, they procure for themselves discord instead of concord; and instead of unanimity, dissension; contending with each other for the mastery. For the wife who surpasses her husband in wealth, in birth, or in friends, is desirous of ruling over him, contrary to the law of nature. But the husband justly resisting this desire of superiority in his wife, and wishing not to be the second, but the first in domestic sway, is unable, in the management of his family, to take the lead.

This being, the case, it happens that not only families, but cities become miserable. For families are parts of cities, while the composition of the whole and the universe derives its subsistence from its parts. It is therefore, reasonable to admit that such as are the parts, such likewise will be the whole and the all which consists of things of this kind. As in fabrics of a primary nature the first structures operate greatly to the good or bad completion of the whole work; as for instance the manner in which the foundation is laid in a house-building, the structure of a keel in ship-building, and the utterance and closing of voice in musical modulation, so the concordant condition of families greatly contributes to the well or ill establishment of a polity.

Those therefore who direct their attention to the propagation of the human species, ought guard against everything which is dissimilar and imperfect; for neither plants nor animals when imperfect are prolific, but their fructification demands a certain amount of time, so that when the bodies are strong and perfect, they may produce seeds and fruits.

Hence it is necessary that boys and girls while they are virgin should be trained up in exercising and proper endurance, and that they be nourished with that kind of food which is adapted to a laborious, temperate, and patient life.

Moreover, in human life there are many things of such a kind that it is better for the knowledge of them to be deferred, for a certain time. Hence a boy should be so tutored as not to seek after venereal pleasure before he is twenty years of age, and then should rarely engage in them. This however will take place if he conceives that a good habit of body and continence are beautiful and honorable.

The following laws should be taught in Grecian cities: that connection with a mother, or a daughter, or a sister, should not be permitted either in temples or in a public place; for it would be well to employ numerous impediments to this energy.

All natural connexions, should be prevented, especially those attended with wanton insolence. But such as harmonize with nature should be encouraged, such as are effected with temperance for the purpose of producing a temperate and legitimate offspring.

Again, those who intend to beget children should providentially attend to the welfare of their future offspring. A temperate and salutary diet therefore is the first and greatest thing to be considered by the would be begetter; so that he should neither be filled with unseasonable food, nor become intoxicated, nor subject himself to any other perturbation which may injure the body -habits. But above all things he should be careful that the mind in the act of copulation should remain in a tranquil state, for bad seed is produced from depraved, discordant and turbulent habits.

With all possible earnestness and attention, we should endeavor that children be born elegant and graceful, and that when born, they should be well educated. For it is foolish that those who rear horses, birds or dog should, with the utmost diligence render the breed perfect, and from proper food, and when it is proper; and likewise consider how they ought to be disposed when they copulate with each other, that the offspring be not the result of chance; while men are inattentive to their progeny, begetting them by chance; and when begotten, should neglect both their food and education. It is the disregard of these that causes all the vice and depravity, since those born thus will resemble cattle, ignoble and vile.

BIOGRAPHY OF PHILOLAUS

BY DIOGENES LAERTES

Philolaus of Crotona, a Pythagorean, was he from whom Plato, in some of his Letters, begged Dio to purchase Pythagorean books. He died under the accusation of having had designs on the tyranny. I have made about him the following epigram:

"I advise everybody to take, good care to avoid suspicion; even if you are not guilty, but seem so, you are ruined. That is why Crotona, the homeland of Philolaus, destroyed him, because he was suspected of wishing to establish autocracy."

He teaches that all things are produced by necessity and harmony, and he is the first who said that the earth has a circular movement; others however insist this was due to Hicetas of Syracuse.

He had written a single book which the philosopher Plato, visiting Dionysius in Sicily, bought according to Hermippus, from Philolaus's parents, for the sum of 40 Alexandrian minae, whence he drew his Timaeus. Others state that he received it as a present for having obtained the liberty of one of Philolaus's disciples, whom Dionysius had imprisoned. In his *Homonyms* Demetrius claims that he is the first of the Pythagorean philosophers who made a work on nature public property. This book begins as follows:

"The world's being is the harmonious compound of infinite and finite principles; such is the totality of the world and all it contains."

FRAGMENTS OF PHILOLAUS

From Boeckh

1. (Stob.21.7; Diog..#.8.85) The world's nature is a harmonious compound of infinite and finite elements; similar is the totality of the world in itself, and of all it contains. b. All beings are necessarily finite or infinite, or simultaneously finite and infinite; but they could not all be infinite only.

2. How, since it is clear that the beings can not be formed neither of elements that are all infinite, it is evident that the world in its totality, and its included beings are a harmonious compound of finite and infinite elements. That can be seen in works of art. Those that are composed of finite elements, are finite themselves, those that are composed of both finite and infinite elements, are both finite and infinite; and those composed of infinite elements, are infinite.

2. All things, at least those we know contain number; for it is evident that nothing whatever can either be thought or known, without number. Number has two distinct kinds: the odd and the even, and a third, derived from a mingling of the other two kinds, the even-odd. Each of its subspecies is susceptible of many very numerous varieties; which each manifests individually.

3. The harmony is generally the result of contraries; for it is the unity of multiplicity, and the agreement of discordances. (Nicom.Arith.2:509)

4. This is the state of affairs about nature and harmony. The essence of things is eternal; it is a unique and divine nature, the knowledge of which does not belong to man. Still it would not be possible that any of the things that are, and are known by us, should arrive to our knowledge, if this essence was not the internal foundation of the principles of which the world was founded, that is, of the finite and infinite elements. How since these principles are not mutually similar, neither of similar nature, it would be impossible that the order of the world should have been formed by them, unless the harmony intervened, in any manner whatever. Of course, the things that were similar and of similar nature, did not need the harmony; but the dissimilar things, which have neither a similar nature, nor an equivalent function, must be organized by the harmony, if they are to take their place in the connected totality of the world.

5. The extent of the harmony is a fourth, plus a fifth. The fifth is greater than the fourth by nine eighths; for from the lowest string to the second lowest, there is a fourth; and from this to the next, a fifth; but from this to the next, or "third," a fourth; and from this "third" to the lowest, a fifth. The interval between the second lowest and the "third" (from the top) is nine eighths; the interval of the fourth, is four thirds; that of the fifth, three halves; that of the octave, the double relation. Thus the harmony contains five nine-eighths plus two sharps; the fifth, three nine eighths, plus one sharp; the fourth two nine-eighths, plus one.

6. (Boethius, Music, 3:5). Nevertheless the Pythagorean Philolaus has tried to divide the tone otherwise; his tone's starting-point is the first uneven number which forms a cube, and you know that the first uneven number was an object of veneration among these Pythagoreans. Now the first odd number is three; thrice three are nine, and nine times three is 27, which differs from the number 24 by the interval of one tone, and differs from it by this very number 3. Indeed, 3 is one eighth of 24, and this eighth part of 24 added to 24 itself, produces 27, the cube of 3. Philolaus divides this number 27 in two parts, the one greater than half, which he calls apotome; the other one smaller than half he calls sharp; but which latterly has become known as minor half-tone. He supposes that this [sharp] contains thirteen unities, because 13 is the difference between 256 and 243, and that this [same] number is the sum of 9, 3, and unity, in which the unity plays the part of the period, 3 of the first odd line, and 9 of the first odd square. After having, for these reasons, expressed by 13 the sharp, which is called a semi-tone, out of 14 unities he forms, the other part of the number 27 which he calls apotome, and as the difference between 13 and 14 is the unity, he insists that the unity forms the coma, and that 27 unities form an entire tone, because 27 is the difference between 215 and 243, which are distant by one tone.

7. (Boethius, Music, 3:8). These are the definitions that Philolaus has given of these intervals, and of still smaller intervals. The coma, says he, is the interval whose eighth-ninths relation exceeds the sum of two sharps, namely, the sum of two minor semi-tones. The *schisma* is half the comma, the *diaschisma* is half the sharp, namely, of the minor semi-tone.

8. (Claudius Namert.de Stat. anim.2:3) Before treating of the substance of the soul, Philolaus, according to geometrical principles, treats of music, arithmetic, measures, weights, numbers, insisting that these are the principles which support the existence of the Universe.

9. (Nicom.. Arith.2:p.72) Some, in this following Philolaus, think that this kind of a proportion is called harmonic, because it has the greatest analogy with what is called geometrical harmony; which is the cube, because all its dimensions are mutually equal, and consequently in perfect harmony. Indeed this proportion is revealed in all kinds of cubes; which has always 12 sides, 8 angles, and 6 surfaces. b.(Cassiodorus, Exp.in Ps.9,p.36) The number 8, which the arithmeticians call the first actual square, has been named, by the Pythagorean Philolaus the name of geometrical harmony, because he thinks he recognizes in it all the hamonic relations.

10. (Stob. Eclogl.1:5:7:p.360) The world is single; it began to form from the centre outwards. Starting from this centre, the top is entirely identical to the base; still you might say that what is above the centre is opposed to what is below it; for, for the base, lowest point would be the centre, as for the top, the highest point would still be the centre; and likewise for the other parts; in fact, in respect to the centre, each one of the opposite points is identical, unless the whole be moved. b.(Stob.Ecl.l:2l:3:p.468) The prime composite, the One placed in the centre of the sphere is called Hestia.

11. a. (Stob.Ecl.l:22:l:p.488) Philolaus has located the fire in the middle, the centre; he calls it Hestia, of the All, the house [policeipest] of Jupiter, and the mother of the Gods, the altar, the link, the measure of nature. Besides, he locates a second fire, quite at the top, surrounding the world. The centre, says he, is by its nature the first; around it, the ten different bodies carry out their choric dance; these are, the heaven, the planets, lower the sun, and below it the moon ; lower the earth, and beneath this, the anti-earth (a body invented by the Pythagoreans, says Aristotle, Met i: 5) then beneath these bodies the fire of Hestia, in the centre, where it maintains order. The highest part of the Covering, in which he asserts that the elements exist in a perfectly pure condition, is called Olympus, the space beneath the revolutionary circle of Olympus, and where in order are disposed the five planets, the sun and moon, forms the Cosmos world; finally, beneath the latter is the sublunar region, which surrounds the earth, where are the generative things susceptible to change; that is the heaven. The order which manifests in the celestial phenomena is the object of science; the disorder which manifests in the things of becoming, is the object of virtue; the former is perfect, the latter is imperfect. b. (Plut. Plac.Phil.3:ll). The Pythagorean Philolaus located the fire in the centre, it is the Hestia of the All, then the Anti-earth, then the earth we inhabit, placed opposite the other, and moving circularly; which is the cause that its inhabitants are not visible to ours. c. (Stob.Ecl.l:21:6:p.452). The directing fire, [of] Philolaus, is in the entirely central fire; which the demiurge has placed as a sort of keel [to] serve as foundation to the sphere of the All.

12. (Plut.Plac.Phil.2:5). Philolaus explains destruction by two causes; one is the fire which descends from heaven, the other is the water of the noon, which is driven away therefrom by the circulation of air; the loss of these two stars nourish the world.

13. (Diog.Laert.8:85). Philolaus was the first who said the world moves in a circle; others attribute it to Hivatas of Syracuse. b. (Plut.Plac.Philos.3:7). Some insist that the earth is immovable but the Pythagorean Philolaus says that it moves circularly around the central fire, in an oblique circle like the sun and moon.

14. (Stob.Ecl.l:25:3:p.530) The Pythagorean Philolaus says that the sun is a vitrescent body which receives the light reflected by the fire of the Cosmos, and sends it back to us, after having filtered them, light and heat; so that you might say that there are two suns, the body of the fire which is in the heaven, and the igneous light which emanates therefrom, and reflects itself in a kind of a mirror. Perhaps we might consider as a third light that which, from the mirror in which it reflects, and falls back on us in dispersed rays.

15. (Stob.Eclog.1:26: l:p. 562) Some Pythagoreans, among whom is Philolaus, pretend that the moon's resemblance to the earth consists in its surface being inhabited, like our earth; but by animals and vegetation larger and more beautiful; for the lunar animals are fifteen times larger than ours, and do not evacuate excreta. The day is also fifteen times as long. Others pretend that the apparent form of the noon is only the reflection of the sea, which we inhabit, which passes beyond the circle of fire.

16. (Censorinus, de Die Natal.18). According to the Pythagorean Philolaus there is a year composed of 59 years and 21 intercalary months; he considers that the natural year has 364 and a half days.

17. (Iambl.ad Nicon.Arith.11). Philolaus says that number is the sovereign and autogenic force which maintains the eternal permanence of cosmic things.

18. (Stob.1:3:8). The power, efficacy and essence of number is seen in the decad; it is great, it realizes all its purposes, it is the cause of all effects; the power of the *decad* is the principle and guide of all life, divine, celestial or human into which it is insinuated; without it everything is infinite, obscure, and furtive. Indeed it is the nature of number which teaches us comprehension, which serves us as guide, which teaches us all things, which would remain impenetrable and unknown for every man, for there is nobody who could get so clear a notion about it, things in themselves, neither in their relations, if there was no number or number-essence. By means of sensation, number instills a certain proportion, and thereby establishes among all things harmonic relations, analogous to the nature of the geometric figure called the *gnomon*; it incorporates intelligible reasons of things, separates them, inidividualises them, both in finite and infinite things. And it is not only in matters pertaining to genii or gods that you may see the force manifested by the nature and power of number, but it is in all its works, in all human thoughts, everywhere indeed, and even in the production of arts and music. The nature of number and harmony are numberless, for what is false has no part in their [-------]; for the principle of error and envy is thoughtless, irrational, infinite nature. Never could error slip into number; for its nature is hostile thereto. Truth is the proper, innate character of number.

b. (Theologoumena, 61). The decad is also named Faith because according to Philolaus, it is by the *decad* and its elements, if utilized energetically and without negligence, that we arrive at a solidly grounded faith about beings. It is also the source of memory, and that is why the *Monad* has been called (Mnemosyne?).

c. (Theon of Smyrna, Platon.Nemn.p.49) The [*Tetractys*] determines every number, including the nature of everything, of the even and the odd, of the mobile and immobile, of good and evil. It has been the subject of long discussions by Archytas, and of Philolaus, in his work on nature.

d. (Lucien, Pro. Laps. Inter. Salut. 5.) Some called the *Tetractys* the great oath of the Pythagoreans, because they considered it the perfect number, or even because it is the principle of health; among them is Philolaus.

19. (Theon of Smyrna, Plat. Math. 4.). Archytas and Philolaus use the terms *monad* and unity interchangeably.

b. (Syrianus, sub init, Comment. in Arist. Net. I.xiv?). You must not suppose that the philosophers begin by principles supposed to be opposite; they know the principle above these two elements, as Philolaus acknowledges when saying that it is God who hypostasizes the finite and the infinite. He shows that it is by the limit, that every coordinate series of things further approaches Unity, and that it is by infinity that the lower series is produced. Thus even above these two principles they posited the unique and separate cause distinguished by all of its excellence. This is the clause which Archinetus called the cause before the cause and which Philolaus vehemently insists is the principle of all, and of which Brontinus says that in power and dignity it surpasses all reason and essence.

c. (Iambl. ad Nicom. Arith. p109). In the formation of square numbers by addition, unity is as it were the starting-post from which one starts, and also the end whither one returns; for if one places the numbers in the form of a double procession, and you see them grow from unity to the root of the square, and the root is like the turning-point where the horses turn to go back through similar numbers to unity, as in the square of 5.

For example:

1-----2-----3-----4
-----5 = added, 25
1-----2-----3-----4

It is not the same with rectangular numbers; if, just as if in the *gnomon*, one adds to any number the sum of the even, then the number two will alone seem to receive and stand addition and without the number two it will not be possible to produce rectangular numbers. If you set out the naturally increasing series of numbers in the order of the double race-track, then unity, being the principle of everything according to Philolaus (for it is he who said, "unity the principle of everything"), will indeed present itself as the barrier, the starting point which produces the rectangular numbers, but it will not be the goal or limit where the series returns, and comes back; it is not unity, but the number 2 which will fulfill this function. Thus:

6 1------2------3-----4 ---------- 5= 24 4 --------2-----3-----4 ----------

d. (Ohilo, Mundi Opif .24). Philolaus confirms what I have just said by the following words; "He who commands and governs everything is a God who is single, eternally existing, immutable, self-identical, different from other things.

e. (Athenag.Legat.pro Christo). Philolaus says that all things are by God kept as if in captivity, and thereby implies, that He is single and superior to matter.

20. (Proclus, ad Euclid. Elem.I.33). Even among the Pythagoreans we find different angles consecrated to the different divinities, as did Philolaus, who devoted to some the angle of the triangle, to others the angle of the rectangle, to others other angles, and sometimes the same to several. The Pythagoreans say that the triangle is the absolute principle of generation of begotten things, and of their form; that is why Timaeus says that the reasons of physical being, and of the regular formation of the elements are triangular; indeed, they have the three dimensions, in unity they gather the elements which in themselves are absolutely divided and changing; they are filled with the infinity characteristic of matter, and above the material beings they form bonds that indeed are frail. That is why triangles are bounded by straight lines and are [have] angles which unite the lines, and are their [ends]. Philolaus was therefore right in devoting the angle of the triangle to four divinities, [Cronos], Hades, Mars and Bacchus, under these names combining the fourfold disposition of the elements, which refers to the superior part of the Universe, starting from the sky, or sections of the zodiac. Indeed, Cronos presides over everything humid and cold essence; Mars, over everything fiery; Hades contains everything terrestrial, and Dionysius directs the generation of wet and warm things, symboled by wine, which is liquid and warm. These four divinities divide their secondary operations, but they remain united; that is why Philolaus, by attributing to them one angle only, wished to express this power of unification.

The Pythagoreans also claim that, in preference to the quadrilateral, the tetragone bears the divine impress; and by it they express perfect order....For the property of being straight imitates the power of immutability; and equality represents that of permanence; for motion is the result of inequality; and rest, that of equality. Those are the causes of the organisation of the being that is solid in its totality, and of its pure and immovable essence. They were therefore right to express it symbolically by the figure the tetragon. Besides, Philolaus, with another stroke of genius, calls the angle of the tetragon. that of Rhea, of Dimeter, and of Hestia...For considering the earth as a tetragon, and noting that this element possesses the property of continuousness, as we learned it from Timaeus, and the earth receives all that drips from the divinities and also the generative powers that they contain, he was right in consecrating the angle of the tetragon to these divinities which procreate life. Indeed, some of them call the earth Hestia and Demeter, and claim that it partakes of Rhea, in its entirety, and that Rhea contains all the begotten causes. That is why, in obscure language, he says, that the angle of the tetragon contains the single power which produces the unity of these divine creations.

And we must not forget that Philolaus assigns the angle of the triangle to four divinities, and the angle of the tetragon to three, thereby indicating their penetrative faculty, whereby they influence each other mutually; showing how all things participate in all things, the odd things in the even and the even in the odd. The *triad* and the *tetrad*, participating in the generative and creative beings, contain the whole regular organization of begotten beings. Their product is the *dodecad*, which ends in the single monad, the sovereign principle of Jupiter; for Philolaus says that the angle of the *dodecagon* belongs to Jupiter, because in unity Jupiter contains the entire of the *dodecad*.

21. (Theolog.Arithm. p.56). After the mathematical magnitude which by its three dimensions or intervals realizes the number four, Philolaus shows us the being manifesting in number five quality and color, in the number six the soul and life; in the number seven, reason, health, and

what he calls light; then he adds that love, friendship, prudence, and reflexion are communicated to beings by the number eight.

b. (Theolog.Arithm. p.22). There are four principles of the reasonable animal, as Philolaus in his work on Nature, the skull, the heart, the navel, and the sexual organs. The head is the seat of reason, the heart, that of the soul or life, and sensation; the navel, the principle of the facility of striking roots and reproducing the first being; the sexual organs, of the faculty of projecting the sperm, and procreating. The skull contains the principle of man, the heart of the animal, the navel that of the plant, the sexual organs that of all living beings, for these grow and produce offspring.

c. (Stob.Edog.Physic.l:2:3: p.l0). There are bodies five in the sphere; fire, water, earth, air; and the circle of the sphere, which makes the fifth.

22. (Stob.Ecog.l:2:2: p.418). From the Pythagorean Philolaus, drawn from his book *On the [Soul]*. He insists that the world is indestructible. Here is what he says in his book *On the Soul*. That is why the world remains eternally, because it cannot be destroyed by any other, nor spontaneously destroy itself. Neither within it, nor without it can be found a force greater than itself; able to destroy it. The world has existed from all eternity, and will remain eternally, because it is single, governed by a principle whose nature is similar to its own, and whose force is omnipotent and sovereign. Besides, the single world is continuous, and endowed with a natural respiration, moving eternally in a circle, having the principle of motion and change; one of its parts is immovable, the other is changing; the immovable part extends from the soul to the moon, that embraces everything, to the moon; and the changing part from the moon to the earth; or, since the mover has been acting since eternity, and continues his action eternally, and since the changeable part receives its manner of being from the Mover who acts thereon, it necessarily results thence that one of the parts of the World ever impresses motion, and that the other ever receives it passively; the one is entirely the domain of reason and the soul, the other of generation and change; the one is anterior in power, and superior, the other is posterior and subordinate. The composite of these two things, the divine eternally in motion and of generation ever changing; is the World. That is why one is right in saying that the World is the eternal energy of God, and of becoming which obeys the laws of changing nature. The one remains eternally in the same state, self-identical, the remainder constitutes the domain of plurality, which is born and perishes. But nevertheless the things that perish save their essence and form, thanks to generation, which reproduces the identical form of the father who has begotten and fashioned them.

23. (Claudian Mamort. De Statu. Anim.2: p.7). The soul is introduced and associated with the body by number and by a harmony simultaneously immortal and incorporeal....The soul cherishes its body, because without it the soul cannot feel; but when death has separated the soul therefrom, the soul lives an incorporeal existence in the cosmos.

b. (Macrob. Dream of Scipio, I:04). Plato says that the soul is a self-moving essence; Xenocrates defines the soul as a self-moving number; Aristotle calls it an *entelechy*; and Pythagoras and Philolaus, a harmony.

c. (Olympiod. ad Plat. Phaed. p 150). Philolaus opposed suicide, because it was a Pythagorean precept not to lay down the burden but to help others carry theirs; namely, that you must assist, and not hinder it.

d. (Clem.Strom. 3: p.433). It will help us to remember the Pythagorean Philolaus's utterance that the ancient theologians and divines claimed that the soul is bound to the body as a punishment, and is buried in it as in a tomb.

24. (Arist.Eth.Eud. 2:8). As Philolaus has said, there are some reasons stronger than us. b. (Iambl. ad Nicom. Arithm. 1:25). I shall later have a better opportunity to consider how, in raising a number to its square, by the position of the simple component unities, we arrive at [very] evident propositions, naturally, and not by any law, as says Philolaus.

25. (Sext. Empir, Adv.Math. 7:92: p. 388). Anaxagoras has said how reason in general is the faculty discerning and judging; the Pythagoreans also agree that it is Reason, not reason in general, but the Reason that develops in men by the study of mathematics, as Philolaus used to say and insist that if this Reason is capable of undestanding All, it is only that its essence is kindred with this nature, for it is in the nature of things that the similar be understood by the similar.

26. (Laurent.Lydus,de Mens.p 16; Cedrenus --; 169b). Philolaus was therefore right in calling it a *decad*, because it receives (pun) the Infinite, and Prpheus was right in calling it the branch, because it is the branch from which issue all the numbers, as do many branches.

b. (Cedrenus, l.p72). Philolaus was therefore right to say that the number seven was motherless.

c. (Cedrenus, l.p.23). Philolaus was therefore right to call the spouse of Kronos, the *Dyad*.

Quotations by Pythagoras

Number is the ruler of forms and ideas, and the cause of gods and demons.
Iamblichus

Every man has been made by God in order to acquire knowledge and contemplate.

Geometry is knowledge of the eternally existent.

Number is the within of all things.

There is geometry in the humming of the strings.

Time is the soul of this world.

Number rules the universe.

Bibliography

Books

The Complete Pythagorus

Papyri Graecae Magicae

Websites

Pythagorus and the Pythagoreans
http://www.math.tamu.edu/~dallen/history/pythag/pythag.html

Pythagoreanism
www.themystica.com/mystica/articles/p/**pythagoreanism**.html

Apollonius of Tyana - Wikipedia
http://www.experiencefestival.com/a/Apollonius_of_Tyana/id/419266

Tetraktys - Wikipedia
http://www.experiencefestival.com/a/Tetractys/id/578326

Pentagram – Wikipedia
http://www.experiencefestival.com/a/Pentagram/id/1896621

Quincunx – Wikipedia
http://www.experiencefestival.com/a/Quincunx/id/553000

Archytas – Wikipedia
http://www.experiencefestival.com/a/Archytas/id/419319

Aristoxenus - Wikipedia
http://www.experiencefestival.com/a/Aristoxenus/id/1918165

The Golden Verses of Pythagorus
http://users.ucom.net/~vegan/